Mount Abu Public School
H-Block, Sector-18, Rohini, New Delhi-110085

SUBJECT:- MATHEMATICS

CLASS-XII

Week : 11 January 2021 to 16 January 2021

CHAPTER-1 : RELATIONS AND FUNCTIONS

Link for the chapter: https://ncert.nic.in/textbook.php?lemh1=1-6

Sub-Topics:

- Relations
- Types of relations
- Functions
- Types of functions

Learning Outcomes:

Each student will be able to:

- Define relations
- Identify types of relations
- Define functions
- Identify types of functions

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

MAPS

DAY-1

LESSON DEVELOPMENT

Relation:

A relation R from set X to a set Y is defined as a subset of the cartesian product X × Y. We can also write it as $R \subseteq \{(x, y) \in X \times Y : xRy\}$.

Note: If n(A) = p and n(B) = q from set A to set B, then n(A × B) = pq and number of relations = 2^{pq}.

Link:

https://www.youtube.com/watch?v=hV1_wvsdJCE

Types of Relation

Empty Relation: A relation R in a set X, is called an empty relation, if no element of X is related to any element of X,
i.e. $R = \Phi \subset X \times X$

Universal Relation: A relation R in a set X, is called universal relation, if each element of X is related to every element of X,
i.e. $R = X \times X$

Reflexive Relation: A relation R defined on a set A is said to be reflexive, if
$(x, x) \in R, \forall x \in A$ or
$xRx, \forall x \in R$

Symmetric Relation: A relation R defined on a set A is said to be symmetric, if
$(x, y) \in R \Rightarrow (y, x) \in R, \forall x, y \in A$ or
$xRy \Rightarrow yRx, \forall x, y \in R$.

Transitive Relation: A relation R defined on a set A is said to be transitive, if
$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R, \forall x, y, z \in A$
or $xRy, yRz \Rightarrow xRz, \forall x, y, z \in R$.

Equivalence Relation: A relation R defined on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

Equivalence Classes: Given an arbitrary equivalence relation R in an arbitrary set X, R divides X into mutually disjoint subsets A, called partitions or sub-divisions of X satisfying

- all elements of A_i are related to each other, for all i.
- no element of A_i is related to any element of A_j, $i \neq j$
- $A_i \cup A_j = X$ and $A_i \cap A_j = 0$, $i \neq j$. The subsets A_i and A_j are called equivalence classes.

Link:

https://www.youtube.com/watch?v=ItfxKIvq1LI

ASSIGNMENT:

Complete questions from NCERT Exercise 1.1 in your register

DAY-2

LESSON DEVELOPMENT

Function:

Let X and Y be two non-empty sets. A function or mapping f from X into Y written as $f : X \to Y$ is a rule by which each element $x \in X$ is associated to a unique element $y \in Y$. Then, f is said to be a function from X to Y.

The elements of X are called the domain of f and the elements of Y are called the codomain of f. The image of the element of X is called the range of X which is a subset of Y.

Note: Every function is a relation but every relation is not a function.

Link:

https://www.youtube.com/watch?v=V2C-wU5-7NY

Types of Functions

One-one Function or Injective Function: A function $f : X \to Y$ is said to be a one-one function, if the images of distinct elements of x under f are distinct, i.e. $f(x_1) = f(x_2) \Leftrightarrow x_1 = x_2, \forall x_1, x_2 \in X$

A function which is not one-one, is known as many-one function.

Onto Function or Surjective Function: A function f : X → Y is said to be onto function or a surjective function, if every element of Y is image of some element of set X under f, i.e. for every y ∈ y, there exists an element X in x such that f(x) = y.
In other words, a function is called an onto function, if its range is equal to the codomain.

Bijective or One-one and Onto Function: A function f : X → Y is said to be a bijective function if it is both one-one and onto.

Link:

https://www.youtube.com/watch?v=bp46gL3WcyQ

Domain and Range of Some Useful Functions

S.No.	Function	Domain	Range
1.	Polynomial function	R	R, if degree is odd. Subset of R, if degree is even.
2.	Rational function	All real except for which $Q(x)=0$.	Depends on particular rational function.
3.	Exponential function $a^x, a>0$	R	$(0, \infty)$
4.	Logarithmic function $\log_a x, x>0, a>0$ and $a \neq 1$	$(0, \infty)$	R
5.	Identity function $y=x$	R	R
6.	Modulus function $\|x\|$	R	$[0, \infty)$
7.	Signum function $\begin{cases} \frac{\|x\|}{x}, x \neq 0 \\ 0, x=0 \end{cases}$	R	$\{-1, 1\}$
8.	Greatest integer function $[x]$	R	Set of integers (I)

Link:

https://www.youtube.com/watch?v=FdK9_Fp76cw

ASSIGNMENT:

Complete questions from NCERT Exercise 1.2 in your register

EXTRA QUESTIONS

Q1)

A relation R in a set A is called __________, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

(1 mark)

(CBSE 2020)

Q2)

Let N be the set of natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) iff ad = bc for all a, b, c, d $\in$ N. Show that R is an equivalence relation.

(4 mark)

(CBSE 2020)

Q3)

Examine whether the operation * defined on R by a * b = ab + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ?

(2 mark)

(CBSE 2019)

Q4)

Show that the relation R on $\mathbb{R}$ defined as R = {(a, b) : a $\leq$ b}, is reflexive, and transitive but not symmetric.

(4 mark)

(CBSE 2019)

Q5)

Prove that the function f : N $\rightarrow$ N, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of f : N $\rightarrow$ S, where S is range of f.

(4 mark)

(CBSE 2019)

Q6)

Find the identity element in the set Q^+ of all positive rational numbers for the operation $*$ defined by $a * b = \frac{3ab}{2}$ for all $a, b \in Q_+$.

(1 mark)

(CBSE 2018)

Q7)

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

(6 mark)

(CBSE 2018)

Q8)

A binary operation $*$ on the set $A = \{0, 1, 2, 3, 4, 5\}$ is defined as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Write the operation table for $a * b$ in A.

Show that zero is the identity for this operation $*$ and each element 'a' $\neq 0$ of the set is invertible with $6 - a$, being the inverse of 'a'.

(6 mark)

(CBSE 2018)

Q9)

Let $A = \mathbb{Q} \times \mathbb{Q}$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to $*$ on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

(6 mark)

(CBSE 2017)

Q10)

Check whether the function $f: R \to R$ defined as $f(x) = x^3$ is one-one or not.

(1 mark)

(Board SQP 2020-21)

Q11)

How many reflexive relations are possible in a set A whose $n(A) = 3$.

(1 mark)(Board SQP 2020-21)

Q12)

A relation R in the set of real numbers **R** defined as $R = \{(a,b): \sqrt{a} = b\}$ is a function or not. Justify

(1 mark)

(Board SQP 2020-21)

Q13)

An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$

(1 mark)

(Board SQP 2020-21)

Q14)

Check whether the relation R in the set Z of integers defined as R = $\{(a,b) : a + b \text{ is "divisible by 2"}\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0 i.e. $[0]$.

(3 mark)

(Board SQP 2020-21)

DAY-3

CHAPTER-2 : INVERSE TRIGONOMETRY

Link for the chapter: https://ncert.nic.in/textbook.php?lemh1=2-6

Sub-Topics:

- Inverse trigonometric functions
- Principal branch
- Principal values
- Properties of inverse trigonometric functions

Learning Outcomes:

Each student will be able to:

- Define inverse trigonometric functions
- Define principal branch trigonometric functions
- Find principal value of various functions
- Solve questions using properties of inverse trigonometric functions

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

LESSON DEVELOPMENT

Inverse Trigonometric Functions:

Trigonometric functions are many-one functions but we know that inverse of function exists if the function is bijective. If we restrict the domain of trigonometric functions, then these functions become bijective and the inverse of trigonometric functions are defined within the restricted domain. The inverse of f is denoted by 'f^{-1}'.
Let y = f(x) = sin x, then its inverse is x = $\sin^{-1}$ y.

Link:

https://www.youtube.com/watch?v=ADpxUQMCSng

Domain and Range of Inverse Trigonometric Functions

Function	Domain	Range (Principal value branch)
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1} x$	R	$(0, \pi)$
$\operatorname{cosec}^{-1} x$	$R-(-1,1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$
$\sec^{-1} x$	$R-(-1,1)$	$[0, \pi]-\left\{\frac{\pi}{2}\right\}$

$\sin^{-1}(\sin\theta) = \theta;\ \forall\ \theta \in [-\pi2,\pi2]$
$\cos^{-1}(\cos\theta) = \theta;\ \forall\ \theta \in [0, \pi]$

$\tan^{-1}(\tan\theta) = \theta;\ \forall\ \theta\ [-\pi2,\pi2]$
$\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = 0;\ \forall\ \theta \in [-\pi2,\pi2]\ ,\ \theta \neq 0$
$\sec^{-1}(\sec\theta) = \theta;\ \forall\ \theta \in [0, \pi],\ \theta \neq \pi2$
$\cot^{-1}(\cot\theta) = \theta;\ \forall\ \theta \in (0, \pi)$

$\sin(\sin^{-1} x) = x,\ \forall\ x \in [-1, 1]$

$\cos(\cos^{-1} x) = x;\ \forall\ x \in [-1, 1]$

$\tan(\tan^{-1}x) = x,\ \forall\ x \in R$

$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$

$\sec(\sec^{-1} x) = x,\ \forall\ x \in (-\infty, -1] \cup [1, \infty)$

$\cot(\cot^{-1} x) = x,\ \forall\ x \in R$

Note: $\sin^{-1}(\sin\theta) = \theta$; $\sin^{-1} x$ should not be confused with $(\sin x)^{-1} = 1\sin x$ or $\sin^{-1} x = \sin^{-1}(1x)$ land similarly for other trigonometric functions.

The value of an inverse trigonometric function, which lies in the range of principal value branch, is called the principal value of the inverse trigonometric function.

Note: Whenever no branch of an inverse trigonometric function is mentioned, it means we have to consider the principal value branch of that function.

MAPS

Properties of Inverse Trigonometric Functions

(a) (i) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x; x \ge 1 \text{ or } x \le -1$ (ii) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x;\ x \ge 1 \text{ or } x \le -1$

(iii) $\tan^{-1}\left(\frac{1}{x}\right) = \begin{cases} \cot^{-1} x;\ x > 0 \\ -\pi + \cot^{-1} x;\ x < 0 \end{cases}$

(b) (i) $\sin^{-1}(-x) = -\sin^{-1} x; x \in [-1, 1]$ (ii) $\tan^{-1}(-x) = -\tan^{-1} x; x \in R$

(iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x; |x| \ge 1$

(c) (i) $\cos^{-1}(-x) = \pi - \cos^{-1} x; x \in [-1, 1]$ (ii) $\sec^{-1}(-x) = \pi - \sec^{-1} x; |x| \ge 1$

(iii) $\cot^{-1}(-x) = \pi - \cot^{-1} x; x \in R$

(d) (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; x \in [-1, 1]$ (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}; x \in R$

(iii) $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}; |x| \ge 1$

(e) (i) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); xy < 1$

(ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right); xy > -1$

(f) (i) $2\tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right); |x| \le 1$ (ii) $2\tan^{-1} x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right); x \ge 0$

(iii) $2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right); -1 < x < 1$

(iv) $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2}); \frac{-1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

(v) $2\cos^{-1} x = \sin^{-1}(2x\sqrt{1-x^2}); \frac{-1}{\sqrt{2}} \le x \le 1$ or $2\cos^{-1} x = \cos^{-1}(2x^2 - 1); 0 \le x \le 1$

(g) (i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

(iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

(iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2})$

(h) (i) $3\sin^{-1} x = \sin^{-1}(3x - 4x^3); \frac{-1}{2} \le x \le \frac{1}{2}$

(ii) $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x); \frac{1}{2} \le x \le 1$

(iii) $3\tan^{-1} x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right); \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

(i) (i) $\sin^{-1} x = \cos^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

(ii) $\cos^{-1} x = \sin^{-1}(\sqrt{1-x^2}) = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

(iii) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Link:

https://www.youtube.com/watch?v=Mh_vfr_1YiU

Following substitutions are used to write inverse trigonometric functions in simplest form:

S.No.	Expression	Substitute
1.	$\sqrt{a^2-x^2}$	$x=a\sin\theta$ or $x=a\cos\theta$
2.	$\sqrt{a^2+x^2}$	$x=a\tan\theta$ or $x=a\cot\theta$
3.	$\sqrt{x^2-a^2}$	$x=a\sec\theta$ or $x=a\operatorname{cosec}\theta$
4.	$\sqrt{a+x}$ or $\sqrt{a-x}$	$x=a\cos\theta$ or $x=a\cos2\theta$
5.	$\sqrt{1+x^2}\pm\sqrt{1-x^2},\sqrt{\frac{1+x^2}{1-x^2}},\sqrt{\frac{1-x^2}{1+x^2}}$	$x^2=\cos^2\theta$
6.	$\sqrt{a^2+x^2}\pm\sqrt{a^2-x^2},\sqrt{\frac{a^2+x^2}{a^2-x^2}},\sqrt{\frac{a^2-x^2}{a^2+x^2}}$	$x^2=a^2\cos2\theta$
7.	$\sqrt{1+x}\pm\sqrt{1-x},\sqrt{\frac{1-x}{1+x}},\sqrt{\frac{1+x}{1-x}}$	$x=\cos2\theta$
8.	$\sqrt{a+x}\pm\sqrt{a-x},\sqrt{\frac{a+x}{a-x}},\sqrt{\frac{a-x}{a+x}}$	$x=a\cos2\theta$

Remember Points

(i) Sometimes, it may happen, that some of the values of x that we find out does not satisfy the given equation.

(ii) While solving an equation, do not cancel the common factors from both sides.

ASSIGNMENT:

Complete questions from NCERT Exercise 2.1 and 2.2 in your register

EXTRA QUESTIONS

Q1)

The principal value of $\tan^{-1}(\tan \frac{3\pi}{5})$ is

(A) $\frac{2\pi}{5}$

(B) $\frac{-2\pi}{5}$

(C) $\frac{3\pi}{5}$

(D) $\frac{-3\pi}{5}$

1 mark (CBSE 2020)

Q2)

Prove that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\cos^{-1} x, \frac{1}{\sqrt{2}} \le x \le 1.$

2 marks (CBSE 2020)

Q3)

Solve : $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}.$

4 marks (CBSE 2019)

Q4)

Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2).$

1 mark (CBSE 2018)

Q5)

Prove that $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right].$

2 marks (CBSE 2018)

Q6)

If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.

4 marks (CBSE 2017)

Q7)

Prove that: [4]

$$\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

4 marks (CBSE 2016)

Q8)

Solve for x : $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

4 marks (CBSE 2016)

Q9)

If $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$, then find x.

4 marks (CBSE 2015)

Q10)

If $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1} x)$, then find x. [4]

4 marks (CBSE 2015)

Q11)

Express $tan^{-1}\left(\frac{cosx}{1-sinx}\right)$, $\frac{-3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

(CBSE Board SPQ 2020-2021)

Week : 18 January 2021 to 23 January 2021

CHAPTER-3 : MATRICES

Link for the chapter: https://ncert.nic.in/textbook.php?lemh1=3-6

Sub-Topics:

- matrices
- Types of matrices
- Operations on matrices

Learning Outcomes:

Each student will be able to:

- Define matrices

- Identify types of matrices
- Perform different operations on matrices
- Solve different problems based on matrices

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

DAY-1

LESSON DEVELOPMENT

A matrix is a rectangular arrangement of numbers (real or complex) which may be represented as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

matrix is enclosed by [ ] or ( ) or | | | |

Link:

https://www.youtube.com/watch?v=JMjbPh1Mjn8

Compact form the above matrix is represented by $[a_{ij}]_{m \times n}$ or $A = [a_{ij}]$.

1. **Element of a Matrix** The numbers a_{11}, a_{12} … etc., in the above matrix are known as the element of the matrix, generally represented as a_{ij} , which denotes element in ith row and jth column.
2. **Order of a Matrix** In above matrix has m rows and n columns, then A is of order m x n.

Types of Matrices

1. **Row Matrix** A matrix having only one row and any number of columns is called a row matrix.

2. **Column Matrix** A matrix having only one column and any number of rows is called column matrix.

3. **Rectangular** Matrix A matrix of order m x n, such that $m \neq n$, is called rectangular matrix.

4. **Horizontal Matrix** A matrix in which the number of rows is less than the number of columns, is called a horizontal matrix.

5. **Vertical Matrix** A matrix in which the number of rows is greater than the number of columns, is called a vertical matrix.

6. **Null/Zero Matrix** A matrix of any order, having all its elements are zero, is called a null/zero matrix. i.e., $a_{ij} = 0, \forall i, j$
7. **Square Matrix** A matrix of order m x n, such that m = n, is called square matrix.

8. **Diagonal Matrix** A square matrix $A = [a_{ij}]_{m \times n}$, is called a diagonal matrix, if all the elements except those in the leading diagonals are zero, i.e., $a_{ij} = 0$ for $i \neq j$. It can be represented as
   $A = \text{diag}[a_{11}\ a_{22} \dots a_{nn}]$
9. **Scalar Matrix** A square matrix in which every non-diagonal element is zero and all diagonal elements are equal, is called scalar matrix. i.e., in scalar matrix
   $a_{ij} = 0$, for $i \neq j$ and $a_{ij} = k$, for $i = j$
10. **Unit/Identity Matrix** A square matrix, in which every non-diagonal element is zero and every diagonal element is 1, is called, unit matrix or an identity matrix.

11. **Upper Triangular Matrix** A square matrix $A = a[_{ij}]_{n \times n}$ is called a upper triangular matrix, if $a[_{ij}], = 0, \forall i > j$.
12. **Lower Triangular Matrix** A square matrix $A = a[_{ij}]_{n \times n}$ is called a lower triangular matrix, if $a[_{ij}], = 0, \forall i < j$.
13. **Submatrix** A matrix which is obtained from a given matrix by deleting any number of rows or columns or both is called a submatrix of the given matrix.

14. **Equal Matrices** Two matrices A and B are said to be equal, if both having same order and corresponding elements of the matrices are equal.

15. **Principal Diagonal of a Matrix** In a square matrix, the diagonal from the first element of the first row to the last element of the last row is called the principal diagonal of a matrix.

16. **Singular Matrix** A square matrix A is said to be singular matrix, if determinant of A denoted by det (A) or |A| is zero, i.e., |A|= 0, otherwise it is a non-singular matrix.

Link:

https://www.youtube.com/watch?v=-hteQKIv-KM

Algebra of Matrices
1. Addition of Matrices
Let A and B be two matrices each of order m x n. Then, the sum of matrices A + B is defined only if matrices A and B are of same order.

If $A = [a_{ij}]_{m \times n}$, $A = [a_{ij}]_{m \times n}$

Then, $A + B = [a_{ij} + b_{ij}]_{m \times n}$

Properties of Addition of Matrices If A, B and C are three matrices of order m x n, then

1. **Commutative Law** $A + B = B + A$
2. **Associative Law** $(A + B) + C = A + (B + C)$
3. **Existence of Additive Identity** A zero matrix (0) of order m x n (same as of A), is additive identity, if
   $A + 0 = A = 0 + A$
4. **Existence of Additive Inverse** If A is a square matrix, then the matrix (- A) is called additive inverse, if
   $A + (-A) = 0 = (-A) + A$
5. **Cancellation Law**
   $A + B = A + C \Rightarrow B = C$ (left cancellation law)
   $B + A = C + A \Rightarrow B = C$ (right cancellation law)

Link:
https://www.youtube.com/watch?v=ZCmVpGv6_1g

2. Subtraction of Matrices

Let A and B be two matrices of the same order, then subtraction of matrices, A – B, is defined as

$A - B = [a_{ij} - b_{ij}]_{n \times n}$,
where $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$

Link:
https://www.youtube.com/watch?v=7jb_AO_hRc8

3. Multiplication of a Matrix by a Scalar

Let $A = [a_{ij}]_{m \times n}$ be a matrix and k be any scalar. Then, the matrix obtained by multiplying each element of A by k is called the scalar multiple of A by k and is denoted by kA, given as
$kA= [ka_{ij}]_{m \times n}$

Properties of Scalar Multiplication

If A and B are matrices of order m x n, then

1. $k(A + B) = kA + kB$
2. $(k_1 + k_2)A = k_1A + k_2A$
3. $k_1k_2A = k_1(k_2A) = k_2(k_1A)$
4. $(-k)A = -(kA) = k(-A)$

Link:

https://www.youtube.com/watch?v=4lHyTQH1iS8

4. Multiplication of Matrices

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} ,$$

Let A = $[a_{ij}]_{m \times n}$ and B = $[b_{ij}]_{n \times p}$ are two matrices such that the number of columns of A is equal to the number of rows of B, then multiplication of A and B is denoted by AB, is given by
where c_{ij} is the element of matrix C and C = AB

Link:
https://www.youtube.com/watch?v=o6tGHLkZvVM
https://www.youtube.com/watch?v=RE-nDY2aWso

Properties of Multiplication of Matrices

1. **Commutative Law** Generally AB ≠ BA
2. **Associative Law** (AB)C = A(BC)
3. **Existence of multiplicative Identity** A.I = A = I.A,
   I is called multiplicative Identity.
4. **Distributive Law** A(B + C) = AB + AC
5. **Cancellation Law** If A is non-singular matrix, then
   AB = AC ⇒ B = C (left cancellation law)
   BA = CA ⇒B = C (right cancellation law)
6. AB = 0, does not necessarily imply that A = 0 or B = 0 or both A and B = 0

Important Points to be Remembered
(i) If A and B are square matrices of the same order, say n, then both the product AB and BA are defined and each is a square matrix of order n.
(ii) In the matrix product AB, the matrix A is called premultiplier (prefactor) and B is called postmultiplier (postfactor).
(iii) The rule of multiplication of matrices is row column wise (or → ↓ wise) the first row of AB is obtained by multiplying the first row of A with first, second, third,… columns of B respectively; similarly second row of A with first, second, third, … columns of B, respectively and so on.

Positive Integral Powers of a Square Matrix
Let A be a square matrix. Then, we can define

1. $A^{n+1} = A^n . A$, where n ∈ N.
2. $A^m . A^n = A^{m+n}$
3. $(A^m)^n = A^{mn}$, ∀ m, n ∈ N

Matrix Polynomial
Let $f(x)= a_0x^n + a_1x^{n-1} -1 + a_2x^{n-2} + \ldots + a_n$. Then
$f(A)= a_0A^n + a_1A^{n-2} + \ldots + a_nI_n$
is called the matrix polynomial.

ASSIGNMENT:

Complete questions from NCERT Exercise 3.1 and 3.2 in your register

DAY-2

LESSON DEVELOPMENT

Transpose of a Matrix

Let $A = [a_{ij}]_{m \times n}$, be a matrix of order m x n. Then, the n x m matrix obtained by interchanging the rows and columns of A is called the transpose of A and is denoted by A' or A^T.

$A' = A^T = [a_{ij}]_{n \times m}$

Link:
https://www.youtube.com/watch?v=g_Rz94DXvNo

Properties of Transpose

1. (A')' = A
2. (A + B)' = A' + B'
3. (AB)' = B'A'
4. (KA)' = kA'
5. $(A^N)' = (A')^N$
6. (ABC)' = C' B' A'

Symmetric and Skew-Symmetric Matrices

1. A square matrix $A = [a_{ij}]_{n \times n}$, is said to be symmetric, if A' = A.
   i.e., $a_{ij} = a_{ji}$, ∀i and j.
2. A square matrix A is said to be skew-symmetric matrices, if i.e., aij = — aji, di and j

Link:

https://www.youtube.com/watch?v=IBgXO5qvbrg

https://www.youtube.com/watch?v=uKPmyG18N7I

https://www.youtube.com/watch?v=S7kQkbpYKJ4

Properties of Symmetric and Skew-Symmetric Matrices

1. Elements of principal diagonals of a skew-symmetric matrix are all zero. i.e., a_{ii} = — a_{ii} 2_{ii} = 0 or a_{ii} = 0, for all values of i.
2. If A is a square matrix, then
   (a) A + A' is symmetric.
   (b) A — A' is skew-symmetric matrix.

3. If A and B are two symmetric (or skew-symmetric) matrices of same order, then A + B is also symmetric (or skew-symmetric).
4. If A is symmetric (or skew-symmetric), then kA (k is a scalar) is also symmetric for skew-symmetric matrix.
5. If A and B are symmetric matrices of the same order, then the product AB is symmetric, iff BA = AB.
6. Every square matrix can be expressed uniquely as the sum of a symmetric and a skew-symmetric matrix.
7. The matrix B' AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric matrix.
8. All positive integral powers of a symmetric matrix are symmetric.
9. All positive odd integral powers of a skew-symmetric matrix are skew-symmetric and positive even integral powers of a skew-symmetric are symmetric matrix.
10. If A and B are symmetric matrices of the same order, then
    (a) AB – BA is a skew-symmetric and
    (b) AB + BA is symmetric.
11. For a square matrix A, AA' and A' A are symmetric matrix.

<u>Link:</u>

https://www.youtube.com/watch?v=hyhktV5pxrE

ASSIGNMENT:

Complete questions from NCERT Exercise 3.3 in your register

EXTRA QUESTION

Q1)

Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

(2 marks) CBSE 2019

Q2)

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations

$x + y + z = 6, x + 2z = 7, 3x + y + z = 12.$

(6 marks)CBSE 2019

Q3)

If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}$ is a matric satisfying $AA' = 9I$, find x.

(1 mark) CBSE 2018

Q5)

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

(4 marks) CBSE 2017

Q6)

Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

(6 marks) CBSE 2017

Q4)

Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

(6 marks) CBSE 2018

Q7)

If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to

(A) I

(B) 0

(C) $I - A$

(D) $I + A$

(1 mark) CBSE 2020

Q8)

If A is a matrix of order 3×2, then the order of the matrix A' is ___________ .

(1 mark) CBSE 2020

Q9)

A square matrix A is said to be skew-symmetric, if ___________ .

(1 mark) CBSE 2020

Q10)

If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$, then find A^{-1} and use it to solve the following system of the equations :

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

(6 marks) CBSE 2020

Q11)

If A and B are matrices of order $3 \times n$ and $m \times 5$ respectively, then find the order of matrix 5A – 3B, given that it is defined.

(1 mark) Board SQP 2020-21

Q12)

Find the value of A^2, where A is a 2×2 matrix whose elements are given by

$$a_{ij} = \begin{cases} 1 & if \;\; i \neq j \\ 0 & if \;\; i = j \end{cases}$$

(1 mark) Board SQP 2020-21

Q13)

If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Hence

Solve the system of equations;

$x - 2y = 10$
$2x - y - z = 8$
$-2y + z = 7$

(5 marks) Board SQP 2020-21

Q14)

Evaluate the product AB, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Hence solve the system of linear equations

$x - y = 3$

$2x + 3y + 4z = 17$

$y + 2z = 7$

(5 marks) Board SQP 2020-21

<u>DAY-3</u>

<u>CHAPTER-4 : DETERMINANTS</u>

<u>Link for the chapter:</u> https://ncert.nic.in/textbook.php?lemh1=4-6

Sub-Topics:

- determinant
- minors
- cofactors

Learning Outcomes:

Each student will be able to:

- Define determinant
- Find the value of determinant of matrices
- Find minors of a given matrix
- Find cofactors of a given matrix

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

MAPS

LESSON DEVELOPMENT

Determinant:

Determinant is the numerical value of the square matrix. So, to every square matrix A = [a_{ij}] of order n, we can associate a number (real or complex) called determinant of the square matrix A. It is denoted by det A or |A|.

Note

(i) Read |A| as determinant A not absolute value of A.

(ii) Determinant gives numerical value but matrix do not give numerical value.

(iii) A determinant always has an equal number of rows and columns, i.e. only square matrix have determinants.

Link:

https://www.youtube.com/watch?v=YFGTpSkfT40

Value of a Determinant

Value of determinant of a matrix of order 2, A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{21} \cdot a_{12}$$

Value of determinant of a matrix of order 3, A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is given by expressing it in terms of second order determinant. This is known as expansion of a determinant along a row (or column).

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \text{(expansion along first row } R_1\text{)}$$

Link:

https://www.youtube.com/watch?v=wrIAFfZBoBM

https://www.youtube.com/watch?v=lhgFVZOgleg

Note

(i) For easier calculations of determinant, we shall expand the determinant along that row or column which contains the maximum number of zeroes.

(ii) While expanding, instead of multiplying by $(-1)^{i+j}$, we can multiply by +1 or -1 according to as (i + j) is even or odd.

Let A be a matrix of order n and let $|A| = x$. Then, $|kA| = k^n |A| = k^n x$, where n = 1, 2, 3,…

Minor:

Minor of an element ay of a determinant, is a determinant obtained by deleting the ith row and jth column in which element ay lies. Minor of an element a_{ij} is denoted by M_{ij}.
Note: Minor of an element of a determinant of order n(n ≥ 2) is a determinant of order (n – 1).

Cofactor:

Cofactor of an element a_{ij} of a determinant, denoted by A_{ij} or C_{ij} is defined as $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is a minor of an element a_{ij}.
Note

(i) For expanding the determinant, we can use minors and cofactors as

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}A_{11} - a_{12}A_{12} + a_{13}A_{13}$$

(ii) If elements of a row (or column) are multiplied with cofactors of any other row (or column), then their sum is zero.

Link:

https://www.youtube.com/watch?v=KMKd993vG9Q

ASSIGNMENT:

Complete questions from NCERT Exercise 4.1, 4.3,4.4 in your register

Week : 25 January 2021 to 30 January 2021

CHAPTER-4 : DETERMINANTS

Link for the chapter: https://ncert.nic.in/textbook.php?lemh1=4-6

Sub-Topics:

- Singular and non singular matrix
- Adjoint of a matrix
- Inverse of a matrix
- Solving system of linear equations

Learning Outcomes:

Each student will be able to:

- Find adjoint of matrix
- Find inverse of matrix
- Solve system of linear equations

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

DAY-1

LESSON DEVELOPMENT

Singular and non-singular Matrix:

If the value of determinant corresponding to a square matrix is zero, then the matrix is said to be a singular matrix, otherwise it is non-singular matrix, i.e. for a square matrix A, if $|A| \neq 0$, then it is said to be a non-singular matrix and of $|A| = 0$, then it is said to be a singular matrix.

Theorems

(i) If A and B are non-singular matrices of the same order, then AB and BA are also non-singular matrices of the same order.
(ii) The determinant of the product of matrices is equal to the product of their respective determinants, i.e. $|AB| = |A||B|$, where A and B are a square matrix of the same order.

Adjoint of a Matrix: The adjoint of a square matrix 'A' is the transpose of the matrix which obtained by cofactors of each element of a determinant corresponding to that

given matrix. It is denoted by adj(A).
In general, adjoint of a matrix $A = [a_{ij}]_{n\times n}$ is a matrix $[A_{ji}]_{n\times n}$, where A_{ji} is a cofactor of element a_{ji}.

Link:

https://www.youtube.com/watch?v=oHzpMgKuI9Q

Properties of Adjoint of a Matrix
If A is a square matrix of order n × n, then

- $A(\text{adj } A) = (\text{adj } A)A = |A|\, I_n$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj } (A^T) = (\text{adj } A)^T$

The **area of a triangle** whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

NOTE: Since the area is a positive quantity we always take the absolute value of the determinant.

Inverse of a Matrix and Applications of Determinants and Matrix
1. Inverse of a Square Matrix: If A is a non-singular matrix (i.e. $|A| \neq 0$), then

$$A^{-1} = \frac{1}{|A|}\text{adj}(A)$$

For $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, the inverse is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$, where A_{ij} is the cofactor of A.

Note: Inverse of a matrix, if exists, is unique.

Link:

https://www.youtube.com/watch?v=AMLUikdDQGk

https://www.youtube.com/watch?v=HYWeEx21WWw

Properties of a Inverse Matrix

- $(A^{-1})^{-1} = A$
- $(A^T)^{-1}=(A^{-1})^T$

- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- $\text{adj}(A^{-1}) = (\text{adj } A)^{-1}$

2. Solution of system of linear equations using inverse of a matrix.

Let the given system of equations be $a_1x + b_1y + c_1z = d_1$; $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$.

We write the following system of linear equations in matrix form as AX = B, where

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}.$$

Case I: If $|A| \neq 0$, then the system is consistent and has a unique solution which is given by $X = A^{-1}B$.

Case II: If $|A| = 0$ and $(\text{adj } A)B \neq 0$, then system is inconsistent and has no solution.

Case III: If $|A| = 0$ and $(\text{adj } A)B = 0$, then system may be either consistent or inconsistent according to as the system have either infinitely many solutions or no solutions

Link:

https://www.youtube.com/watch?v=NNmiOoWt86M

https://www.youtube.com/watch?v=a2z7sZ4MSqo

ASSIGNMENT:

Complete questions from NCERT Exercise 4.5 , 4.6 in your register

EXTRA QUESTIONS:

Q1)

If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A'|$ is

(A) -10

(B) 10

(C) -40

(D) 40

(1 mark) CBSE 2020

Q2)

If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find A (adj A).

(1 mark) CBSE 2020

Q3)

If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of |B|.

(1 mark) CBSE 2019

Q4)

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k.

(2 marks) CBSE 2018

Q5)

If for any 2×2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.

(1 mark) CBSE 2017

Q6)

Given that A is a square matrix of order 3×3 and |A| = - 4. Find |adj A|

(1 mark) Board SQP 2020-21

Q7)

Let A = $[a_{ij}]$ be a square matrix of order 3×3 and |A|= -7. Find the value of

$$a_{11}\,A_{21} + a_{12}A_{22} + a_{13}\,A_{23}$$

where A_{ij} is the cofactor of element a_{ij}

(1 mark) Board SQP 2020-21

Q8)

If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of |A|.

(2 marks) Board SQP 2020-21

Q9)

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.

Hence find A^{-1}.

(2 marks) Board SQP 2020-21

CHAPTER-5 : CONTINUITY AND DIFFERENTIABILITY

Link for the chapter: https://ncert.nic.in/textbook.php?lemh1=5-6

Sub-Topics:

- continuity
- differentiability
- differentiation of implicit functions
- differentiation in parametric form
- logarithmic differentiation
- second order derivative

Learning Outcomes:

Each student will be able to:

- check the continuity of functions
- check differentiability of functions
- differentiate functions using appropriate methods
- find second order derivative of functions

Teaching Aids Used:

Presentation of E-lesson, YouTube videos by screen sharing, white board and marker using laptop/mobile

GUIDELINES:

Dear students,

Kindly read the content given below and view the links shared for better understanding.

Solve the given questions in math notebook.

DAY-2

LESSON DEVELOPMENT

Continuity at a Point:

A function f(x) is said to be continuous at a point x = a, if

Left hand limit of f(x) at(x = a) = Right hand limit of f(x) at (x = a) = Value of f(x) at (x = a)

i.e. if at x = a, LHL = RHL = f(a)

where, LHL = $\lim_{x\to a^-} f(x)$ and RHL = $\lim_{x\to a^+} f(x)$

Note: To evaluate LHL of a function f(x) at (x = o), put x = a – h and to find RHL, put x = a + h.

MAPS

Link:
https://www.youtube.com/watch?v=a22gEBQpKpE

Continuity in an Interval:

A function y = f(x) is said to be continuous in an interval (a, b), where a < b if and only if f(x) is continuous at every point in that interval.

- Every identity function is continuous.
- Every constant function is continuous.
- Every polynomial function is continuous.
- Every rational function is continuous.
- All trigonometric functions are continuous in their domain.

Question 1:

Prove that the function $f(x)=5x-3$ is continuous at $x=0, x=-3$ and at $x=5$.

Solution 1:

The given function is $f(x)=5x-3$

At $x=0, f(0)=5\times0-3=3$

$\lim\limits_{x\to0} f(x)=\lim\limits_{x\to0}(5x-3)=5\text{x}0\text{-}3\text{=-}3$

$\therefore \lim\limits_{x\to0} f(x)=f(0)$

Therefore, f is continuous at $\text{x}=0$

At $x=-3, f(-3)=5\text{x}(\text{-}3)-3=-18$

$\lim\limits_{x\to3} f(x)=\lim\limits_{x\to3} f(5x-3)=5\text{x}(\text{-}3)-3=-18$

$\therefore \lim\limits_{x\to3} f(x)=f(-3)$

Therefore, f is continuous at $\text{x}=\text{-}3$

At $x=5, f(x)=f(5)=5\text{x}5-3=25-3=22$

$\lim\limits_{x\to5} f(x)=\lim\limits_{x\to5}(5x-3)=5\text{x}5\text{-}3=22$

$\therefore \lim\limits_{x\to5} f(x)=f(5)$

Therefore, f is continuous at $\text{x}=5$

Question 4:

Prove that the function $f(x)=x^n$ is continuous at $x=n$ is a positive integer.

Solution 4:

The given function is $f(x)=x^n$

It is evident that f is defined at all positive integers, n, and its value at n is n^n.

Then, $\lim\limits_{x\to n} f(n)=\lim\limits_{x\to n} f(x^n)=n^n$

$\therefore \lim\limits_{x\to n} f(x)=f(n)$

Therefore, f is continuous at n, where n is a positive integer.

Question 6:

Find all points of discontinuous of f, where f is defined by

$$f(x)=\begin{cases}2x+3, & if\ x\le 2\\ 2x-3, & if\ x>2\end{cases}$$

Solution 6:

The give function f is $f(x)=\begin{cases}2x+3, & if\ x\le 2\\ 2x-3, & if\ x>2\end{cases}$

It is evident that the given function f is defined at all the points of the real line.

Let c be a point on the real line. Then, three cases arise.

I. $c<2$

II. $c>2$

III. $c=2$

$Case\ (i)\ c<2$

Then, $f(x)=2x+3$

$\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(2x+3)=2c+3$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points, x, such that $x<2$

Case $(ii) c>2$

Then, $f(c)=2c-3$

$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x-3)=2c-3$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x>2$

Case $(iii) c=2$

Then, the left hand limit of f at $x=2$ is,

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(2x+3)=2\text{x}2+3=7$$

The right hand limit of f at $x=2$ is,

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(2x+3)=2\text{x}2-3=1$$

It is observed that the left and right hand limit of f at $x=2$ do not coincide.

Therefore, f is not continuous at $x=2$

Hence, $x=2$ is the only point of discontinuity of f.

Question 7:

Find all points of discontinuity of f, where f is defined by

$$f(x)=\begin{cases}|x|+3, & \text{if } x\le -3\\ -2x, & \text{if } -3<x<3\\ 6x+2, & \text{if } x\ge 3\end{cases}$$

Solution 7:

The given function f is $f(x)=\begin{cases}|x|+3, & \text{if } x\le -3\\ -2x, & \text{if } -3<x<3\\ 6x+2, & \text{if } x\ge 3\end{cases}$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I :

If $c<-3$, then $f(c)=-c+3$

$$\lim_{x\to c} f(x)=\lim_{x\to c}(-x+3)=-c+3$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<-3$

Case II :

If $c=-3$, then $f(-3)=-(-3)+3=6$

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(-x+3)=-(-3)+3=6$$

$$\therefore \lim_{x\to 3^+} f(x)=\lim_{x\to 3^+} f(-2x)=2x(-3)=6$$

$$\therefore \lim_{x\to 3} f(x)=f(-3)$$

Therefore, f is continuous at $x=-3$

Case III :

If $-3<c<3$, then $f(c)=-2c$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to 3c}(-2x)=-2c$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous in $(-3,3)$.

Case IV:

If $c=3$, then the left hand limit of f at $x=3$ is,

$\lim\limits_{x\to 3^-} f(x)=\lim\limits_{x\to 3^-} f(-2x)=-2x3=-6$

The right hand limit of f at $x=3$ is,

$\lim\limits_{x\to 3^+} f(x)=\lim\limits_{x\to 3^+} f(6x+2)=6x3+2=20$

It is observed that the left and right hand limit of f at $x=3$ do not coincide.

Therefore, f is not continuous at $x=3$

Case V:

If $c>3$, then $f(c)=6c+2$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(6x+2)=6c+2$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x>3$

Hence, $x=3$ is the only point of discontinuity of f.

Question 9:

Find all points of discontinuity of f, where f is defined by $f(x)=\begin{cases}\dfrac{x}{|x|}, & if\ x<0\\ -1, if & x\geq 0\end{cases}$

Solution 9:

The given function f is $f(x)=\begin{cases}\frac{x}{|x|}, if\ x<0\\ -1, if\ x\geq 0\end{cases}$

It is known that, $x<0\Rightarrow |x|=-x$

Therefore, the given function can be rewritten as

$$f(x)=\begin{cases}\frac{x}{|x|}, if\ x<0\\ -1, if\ x\geq 0\end{cases}$$

$\Rightarrow f(x)=-1$ for all $x\in \mathbf{R}$

Let c be any real number. Then , $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(-1)=-1$

Also, $f(c)=-1=\lim\limits_{x\to c} f(x)$

Therefore, the given function is continuous function.
Hence, the given function has no point of discontinuity.

Question 18:

For what value of λ is the function defined by $f(x)=\begin{cases}\lambda(x^2-2x), if\ x\leq 0\\ 4x+1, \quad if\ x>0\end{cases}$ continuous at $x=0$? what about continuity at $x=1$?

Solution 18:

The given function f is $f(x)=\begin{cases}\lambda(x^2-2x), if\ x\leq 0\\ 4x+1, \quad if\ x>0\end{cases}$

If f is continuous at $x=0$, then

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-} f(x)=f(0)$

$\Rightarrow \lim\limits_{x\to 0^-} \lambda(x^2-2x)=\lim\limits_{x\to 0^+}(4x+1)=\lambda(0^2-2\text{ x}0)$

$\Rightarrow \lambda(0^2-2\text{ x}0)=4\text{ x}0+1=0$

$\Rightarrow 0=1=0$, which is not possible

Therefore, there is no value of λ for which f is continuous at $x=0$

At $x=1$,

$f(1)=4x+1=4\text{ x }1+1=5$

$\lim\limits_{x\to 1}(4x+1)=4\text{x}1+1=5$

$\therefore \lim_{x\to 1} f(x) = f(1)$

Therefore, for any values of λ, f is continuous at $x = 1$

Question 30:

Find the values of a and b such that the function defined by $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21 & \text{if } x \ge 10 \end{cases}$ is a continuous function.

Solution 30:

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21 & \text{if } x \ge 10 \end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} (5) = \lim_{x\to 2^+} (ax+b) = 5$

$\Rightarrow 5 = 2a + b = 5$

$\Rightarrow 2a + b = 5$ ....(1)

Since f is a continuous at $x = 10$, we obtain

$\lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$

$\Rightarrow \lim_{x\to 10^-} (ax+b) = \lim_{x\to 10^+} (21) = 21$

$\Rightarrow 10a + b - 21 = 21$

$\Rightarrow 10a + b = 21$ .....(2)

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$\Rightarrow a = 2$

By putting a = 2 in equation (1), we obtain

$2 \times 2 + b = 5$

$\Rightarrow 4 + b = 5$

$\Rightarrow b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

ASSIGNMENT:

Complete questions from NCERT Exercise 5.1 in your register

Link:

https://www.youtube.com/watch?v=8srKDVTyi64&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=6

- **Standard Results of Limits**

(i) $\lim\limits_{x \to a} \dfrac{x^n - a^n}{x - a} = na^{n-1}$ (ii) $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$ (iii) $\lim\limits_{x \to 0} \dfrac{\tan x}{x} = 1$

(iv) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x} = 1$ (v) $\lim\limits_{x \to \infty} \dfrac{1}{x^p} = 0, p \in (0, \infty)$ (vi) $\lim\limits_{x \to 0} \dfrac{\log(1 + x)}{x} = 1$

(vii) $\lim\limits_{x \to 0} \dfrac{a^x - 1}{x} = \log_e a$ (viii) $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x}{x} = 1$ (x) $\lim\limits_{x \to 0} (1+x)^{1/x} = e$

(xi) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x} = 0$ (xii) $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x = e$

(xiii) $\lim\limits_{x \to \infty} \sin x = \lim\limits_{x \to \infty} \cos x = \text{lies between} - 1 \text{ to } 1.$

- **Algebra of Continuous Functions**
  Suppose f and g are two real functions, continuous at real number c. Then,

Suppose f and g are two real functions, continuous at real number c. Then,

- f + g is continuous at x = c.
- f – g is continuous at x = c.
- f.g is continuous at x = c.
- cf is continuous, where c is any constant.
- (fg) is continuous at x = c, [provide g(c) ≠ 0]

Suppose f and g are two real valued functions such that (fog) is defined at c. If g is continuous at c and f is continuous at g (c), then (fog) is continuous at c.

If f is continuous, then |f| is also continuous.

Link:

https://www.youtube.com/watch?v=5M5pBVrxbaQ

Differentiability: A function f(x) is said to be differentiable at a point x = a, if
Left hand derivative at (x = a) = Right hand derivative at (x = a)
i.e. LHD at (x = a) = RHD (at x = a), where Right hand derivative, where

Right hand derivative, $Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Left hand derivative, $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$

Note: Every differentiable function is continuous but every continuous function is not differentiable.

Link:

https://www.youtube.com/watch?v=cn1GUBJCMqo&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=1&t=303s

Differentiation: The process of finding a derivative of a function is called differentiation.

Rules of Differentiation
Sum and Difference Rule: Let y = f(x) ± g(x).Then, by using sum and difference rule, it's derivative is written as

$$\frac{dy}{dx} = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x).$$

Product Rule: Let y = f(x) g(x). Then, by using product rule, it's derivative is written as

$$\frac{dy}{dx} = \left[\frac{d}{dx}(f(x))\right] g(x) + \left[\frac{d}{dx}(g(x))\right] f(x).$$

Quotient Rule: Let y = $\frac{f(x)}{g(x)}$; g(x) ≠ 0, then by using quotient rule, it's derivative is written as

$$\frac{dy}{dx} = \frac{g(x) \times \frac{d}{dx}[f(x)] - f(x) \times \frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

Link:

https://www.youtube.com/watch?v=eElNoN2LqKA&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=2

Question 3:

Differentiate the functions with respect of x .

$\sin(ax+b)$

Solution 3:

Let $f(x)=\sin(ax+b), u(x)=ax+b,$ and $v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v(ax+b)=\sin(ax+b)=f(x)$

Thus, f is a composite function of two functions u and v.

Put $t=u(x)=ax+b$

Therefore,

$$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos(ax+b)$$

$$\frac{dt}{dx}=\frac{d}{dx}(ax+b)=\frac{d}{dx}(ax)+\frac{d}{dx}(b)=a+0=a$$

Hence, by chain rule, we obtain

$$\frac{df}{dx}=\frac{dv}{dt}.\frac{dt}{dx}=\cos(ax+b)\cdot a=a\cos(ax+b)$$

Alternate method

$$\frac{d}{dx}[\sin(ax+b)]=\cos(ax+b).\frac{d}{dx}(ax+b)$$

$$=\cos(ax+b).\left[\frac{d}{dx}(ax)+\frac{d}{dx}(b)\right]$$

$$=\cos(ax+b).(a+0)$$

$$=a\cos(ax+b)$$

Question 4:

Differentiate the functions with respect of x.

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

Solution 4:

Let $f(x)=\sec\left(\tan\left(\sqrt{x}\right)\right), u(x)=\sqrt{x},\ v(t)=\tan t$, and $w(s)=\sec s$

Then, $(wovou)(x)=w\left[v(u(x))\right]=w\left[v\left(\sqrt{x}\right)\right]=w\left(\tan\sqrt{x}\right)=\sec\left(\tan\sqrt{x}\right)=f(x)$

Thus, f is a composite function of three functions, u, v and w.

Put $s=v(t)=\tan t$ and $t=u(x)=\sqrt{x}$

Then, $\frac{dw}{ds}=\frac{d}{ds}(\sec s)=\sec s\tan s=\sec(\tan t).\tan(\tan t) \quad [s=\tan t]$

$=\sec(\tan\sqrt{x}).\tan\left(\tan\sqrt{x}\right) \qquad [t=\sqrt{x}]$

$\frac{ds}{dt}=\frac{d}{dt}(\tan t)=\sec^2 t=\sec^2\sqrt{x}$

$\frac{dt}{dx}=\frac{d}{dx}\left(\sqrt{x}\right)=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)=\frac{1}{2}.x^{\frac{1}{2}-1}=\frac{1}{2\sqrt{x}}$

Hence, by chain rule, we obtain

$\frac{dt}{dx}=\frac{dw}{ds}.\frac{ds}{dt}.\frac{dt}{dx}$

$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\text{x } \sec^2\sqrt{x}\text{ x }\frac{1}{2\sqrt{x}}$

$$=\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)$$

$$=\frac{\sec^2\sqrt{x}\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)}{2\sqrt{x}}$$

Alternate method

$$\frac{d}{dx}\left[\sec\left(\tan\sqrt{x}\right)\right]=\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right).\frac{d}{dx}\left(\tan\sqrt{x}\right)$$

$$=\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right).\sec^2\left(\sqrt{x}\right).\frac{d}{dx}\left(\sqrt{x}\right).$$

$$=\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right).\sec^2\left(\sqrt{x}\right).\frac{1}{2\sqrt{x}}$$

$$=\frac{\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right).\sec^2\left(\sqrt{x}\right)}{2\sqrt{x}}$$

Question 7:

Differentiate the functions with respect to x.

$$2\sqrt{\cot\left(x^2\right)}$$

Solution 7:

$$\frac{d}{dx}\left[2\sqrt{\cot\left(x^2\right)}\right]$$

$$=2.\frac{1}{2\sqrt{\cot\left(x^2\right)}}\text{x}\frac{\text{d}}{\text{dx}}\left[\cot\left(x^2\right)\right]$$

$$=\sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\text{x-cosec}^2\left(x^2\right)\text{x}\frac{d}{dx}\left(x^2\right)$$

$$=\sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\text{x}\frac{1}{\sin^2\left(x^2\right)}\text{x}(2x)$$

$$=\frac{-2x}{\sqrt{\cos x^2\sqrt{\sin x^2\sin x^2}}}$$

$$=\frac{-2\sqrt{2}x}{\sqrt{2\sin x^2\cos x^2}\sin x^2}$$

$$=\frac{-2\sqrt{2}x}{\sin x^2\sqrt{\sin 2x^2}}$$

Question 9:

Prove that the function f given by

$f(x)=|x-1|,\ x\in\mathbf{R}$ is not differentiable at $x=1$.

Solution 9:

The given function is $f(x)=|x-1|, x\in\mathbf{R}$

It is known that a function f is differentiable at a point $x=c$ in its domain if both

$\lim\limits_{k\to0^-}\frac{f(c+h)-f(c)}{h}$ and $\lim\limits_{h\to0^+}\frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at $x=1$,

Consider the left hand limit of f at $x=1$

$$\lim_{h\to0^-}\frac{f(1+h)-f(1)}{h}=\lim_{h\to0^-}\frac{f|I+h-1||1-1|}{h}$$

$$=\lim_{h\to0^-}\frac{|h|-0}{h}=\lim_{h\to0^-}\frac{-h}{h} \qquad (h<0\Rightarrow|h|=-h)$$

$$=-1$$

Consider the right hand limit of f at $x=1$

$$\lim_{h\to0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to0^+}\frac{f|I+h-1|-|1-1|}{h}$$

$$=\lim_{h\to0^+}\frac{|h|-0}{h}=\lim_{h\to0^+}\frac{h}{h} \qquad (h>0\Rightarrow|h|=h)$$

$$=1$$

Since the left and right hand limits of f at $x=1$ are not equal, f is not differentiable at $x=1$

ASSIGNMENT:

Complete questions from NCERT Exercise 5.2 in your register

Link:

https://www.youtube.com/watch?v=efmAd2MG4Uo&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=7

Chain Rule: Let y = f(u) and u = f(x), then by using chain rule, we may write

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}, \text{ when } \frac{dy}{du} \text{ and } \frac{du}{dx} \text{ both exist.}$$

Link:

https://www.youtube.com/watch?v=n4N2V6t4o4w&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=3

Question 3:

Find $\frac{dy}{dx}: ax+by^2=\cos y$

Solution 3:

The given relationship is $ax+by^2=\cos y$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}(ax)+\frac{d}{dx}\left(by^2\right)=\frac{d}{dx}(\cos y)$$

$$\Rightarrow a+b\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}(\cos y) \qquad \ldots(1)$$

Using chain rule, we obtain $\frac{d}{dx}\left(y^2\right)=2y\frac{dy}{dx}$ and $\frac{d}{dx}(\cos y)=\sin y\frac{dy}{dx}$ ……….(2)

From (1) and (2), we obtain

$$a+bx\ \ 2y\frac{dy}{dx}=-\sin y\frac{dy}{dx}$$

$$\Rightarrow (2by+\sin y)\frac{dy}{dx}=-a$$

$$\therefore \frac{dy}{dx}=\frac{-a}{2by+\sin y}$$

Question 5:

Find $\frac{dy}{dx}: x^2+xy+y^2=100$

Solution 5:

The given relationship is $x^2+xy+y^2=100$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}\left(x^2+xy+y^2\right)=\frac{d}{dx}(100)$$

[Derivative of constant function is 0]

$$\Rightarrow \frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(xy)+\frac{d}{dx}\left(y^2\right)=0$$

$$\Rightarrow 2x+\left[y.\frac{d}{dx}(x)+x.\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0$$ [Using product rule and chain rule]

$$\Rightarrow 2x+y.1+x.\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$\Rightarrow 2x+y+(x+2y)\frac{dy}{dx}=0$$

$$\therefore \frac{dy}{dx}=-\frac{2x+y}{x+2y}$$

Question 7:

Find $\frac{dx}{dy}: \sin^2 y + \cos xy = \pi$

Solution 7:

The given relationship is $\sin^2 y + \cos xy = \pi$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}\left(\sin^2 y + \cos xy\right) = \frac{d}{dx}(\pi)$$

$$\Rightarrow \frac{d}{dx}\left(\sin^2 y\right) + \frac{d}{dx}\left(\cos xy\right) = 0 \quad .....(1)$$

Using chain rule, we obtain

$$\frac{d}{dx}\left(\sin^2 y\right) = 2\sin y \frac{d}{dx}\left(\sin y\right) = 2\sin y \cos y \frac{dy}{dx} \quad ....(2)$$

$$\frac{d}{dx}\left(\cos xy\right) = -\sin xy \frac{d}{dx}(xy) = -\sin xy\left[y\frac{d}{dx}(x) + x\frac{dy}{dx}\right]$$

$$= -\sin xy\left[y.1 + x\frac{dy}{dx}\right] = -y\sin xy - x\sin xy\frac{dy}{dx} \quad .....(3)$$

From (1) ,(2) and (3), we obtain

$$2\sin y\cos y\frac{dy}{dx} - y\sin xy - x\sin xy\frac{dy}{dx} = 0$$

$$\Rightarrow \left(2\sin y\cos y - x\sin xy\right)\frac{dy}{dx} = y\sin xy$$

$$\Rightarrow \left(\sin 2y - x\sin xy\right)\frac{dx}{dy} = y\sin xy$$

Question 10:

Find $\frac{dx}{dy}: y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Solution 10:

The given relationship is $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

$y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ ....(1)

$\Rightarrow \tan y=\frac{3x-x^3}{1-3x^2}$

It is known that, $\tan y=\frac{3\tan\frac{y}{3}-\tan^3\frac{y}{3}}{1-3\tan^2\frac{y}{3}}$ ......(2)

Comparing equations (1) and (2), we obtain

$x=\tan\frac{y}{3}$

Differentiating this relationship with respect to x, we obtain

$\frac{d}{dx}(x)=\frac{d}{dx}\left(\tan\frac{y}{3}\right)$

$\Rightarrow 1=\sec^2\frac{y}{3}.\frac{d}{dx}\left(\frac{y}{3}\right)$

$\Rightarrow 1=\sec^2\frac{y}{3}.\frac{1}{3}.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{3}{\sec^2\frac{y}{3}}=\frac{3}{1+\tan^2\frac{y}{3}}$

$\therefore \frac{dx}{dy}=\frac{3}{1+x^2}$

Question 12:

Find $\frac{dy}{dx}: y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0<x<1$

Solution 12:

The given relationship is $y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \sin y=\frac{1-x^2}{1+x^2}$

Differentiating this relationship with respect to x, we obtain

$\frac{d}{dx}(\sin y)=\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$ ......(1)

Using chain rule, we obtain

$$\frac{d}{dx}(\sin y)=\cos y.\frac{dy}{dx}$$

$$\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}$$

$$=\sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}}=\sqrt{\frac{4x^2}{(1+x^2)^2}}=\frac{2x}{1+x^2}$$

$$\therefore \frac{d}{dx}(\sin y)=\frac{2x}{1+x^2}\frac{dy}{dx} \qquad \ldots\ldots.(2)$$

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)=\frac{(1+x^2)(1-x^2)'-(1-x^2)(1+x^2)'}{(1+x^2)^2} \qquad \text{[using quotient rule]}$$

$$=\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$=\frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}$$

$$=\frac{-4x}{(1+x^2)^2} \qquad \ldots..(3)$$

From (1),(2), and (3), we obtain

$$\frac{2x}{1+x^2}\frac{dy}{dx}=\frac{-4x}{(1+x^2)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2}{1+x^2}$$

Alternate method

$$y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow \sin y=\frac{1-x^2}{1+x^2}$$

$$\Rightarrow \left(1+x^2\right)\sin y=1-x^2$$

$$\Rightarrow \left(1+\sin y\right)x^2=1-\sin y$$

$$\Rightarrow x^2=\frac{1-\sin y}{1+\sin y}$$

$$\Rightarrow x^2=\frac{\left(\cos\frac{y}{2}-\sin\frac{y}{2}\right)^2}{\left(\cos\frac{y}{2}+\sin\frac{y}{x}\right)^2}$$

$$\Rightarrow x=\frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}$$

$$\Rightarrow x=\frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}$$

$$\Rightarrow x=\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}(x)=\frac{d}{dx}.\left[\tan\left(\frac{\pi}{4}-\frac{y}{2}\right)\right]$$

$$\Rightarrow 1=\sec^2\left(\frac{\pi}{4}-\frac{y}{2}\right).\frac{d}{dx}\left(\frac{\pi}{4}-\frac{y}{2}\right)$$

$$\Rightarrow 1=\left[1+\tan^2\left(\frac{\pi}{4}-\frac{y}{2}\right).\left(-\frac{1}{2}.\frac{dy}{dx}\right)\right.$$

$$\Rightarrow 1=\left(1+x^2\right)\left(-\frac{1}{2}\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dx}{dy}=\frac{-2}{1+x^2}$$

Question 15:

Find $\frac{dy}{dx}$: $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$

Solution 15:

The given relationship is $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$$y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$$

$$\Rightarrow \sec y = \frac{1}{2x^2-1}$$

$$\Rightarrow \cos y = 2x^2 - 1$$

$$\Rightarrow 2x^2 = 1 + \cos y$$

$$\Rightarrow 2x^2 = 2\cos^2\frac{y}{2}$$

$$\Rightarrow x = \cos\frac{y}{2}$$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(\cos\frac{y}{2}\right)$$

$$\Rightarrow 1 = \sin\frac{y}{2}.\frac{d}{dx}\left(\frac{y}{2}\right)$$

$$\Rightarrow \frac{-1}{\sin\frac{y}{2}} = \frac{1}{2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sin\frac{y}{2}} = \frac{-2}{\sqrt{1-\cos^2\frac{y}{2}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

ASSIGNMENT:

Complete questions from NCERT Exercise 5.3 in your register

Link:

https://www.youtube.com/watch?v=PRayfXZZEhc&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=8

Logarithmic Differentiation: Let y = $[f(x)]^{g(x)}$ ..(i)
So by taking log (to base e) we can write Eq. (i) as log y = g(x) log f(x). Then, by using chain rule

$$\frac{dy}{dx} = [f(x)]^{g(x)}\left[\frac{g(x)}{f(x)} f'(x) + g'(x)\log f(x)\right]$$

Link:

https://www.youtube.com/watch?v=4HyUk16-EZo&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=4

DAY-3

LESSON DEVELOPMENT

Question 2:

Differentiating the following $e^{\sin^{-1}x}$

Solution 2:

Let $y = e^{\sin^{-1}x}$

differentiating w.r.t x , we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1}x}\right)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1}x}.\frac{d}{dx}\left(\sin^{-1}x\right)$$

$$\Rightarrow e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{e\sin^{-1}x}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}, x \in (-1,1)$$

Question 4:

Differentiating the following $w.r.t.\ x$: $\sin\left(\tan^{-1} e^{-x}\right)$

Solution 4:

Let $y = \sin\left(\tan^{-1} e^{-x}\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} : \frac{d}{dx}\left[\sin\left(\tan^{-1} e^{-x}\right)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right).\frac{d}{dx}\left(\tan^{-1} e^{-x}\right)$$

$$= \cos\left(\tan^{-1} e^{-x}\right).\frac{1}{1+\left(e^{-x}\right)^2}.\frac{d}{dx}\left(e^{-x}\right)$$

$$= \frac{\cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}.e^{-x}.\frac{d}{dx}(-x)$$

$$= \frac{e^{-x}\cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}\text{x}(-1)$$

$$= \frac{-e^{-x}\cos\left(\tan^{-1} e^{-x}\right)}{1+e^{-2x}}$$

Question 6:

Differentiating the following $w.r.t.\ x$: $e^x + e^{x^2} + ... + e^{x^5}$

Solution 6:

$$\frac{d}{dx}\left(e^x + e^{x^2} + ... + e^{x^5}\right)$$

$$= \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$$

$$= e^x + \left[e^{x^2}\text{x}\frac{d}{dx}\left(x^2\right)\right] + \left[e^{x^3}\text{x}\frac{d}{dx}\left(x^3\right)\right] + \left[e^{x^4}\text{x}\frac{d}{dx}\left(x^4\right)\right] + \left[e^{x^5}\text{x}\frac{d}{dx}\left(x^5\right)\right]$$

$$= e^x + \left(e^{x^2}\text{x}2x\right) + \left(e^{x^3}\text{x}3x^2\right) + \left(e^{x^4}\text{x}4x^3\right) + \left(e^{x^5}\text{x}5x^4\right)$$

$$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$$

Question 8:

Differentiating the following $w.r.t.\ x: l\log(l\log x), x>1$

Solution 8:

Let $y = l\log(l\log x)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left[l\log(l\log x)\right]$$

$$= \frac{1}{l\log x}.\frac{d}{dx}(l\log x)$$

$$= \frac{1}{l\log x}.\frac{1}{x}$$

$$\frac{1}{x\log x}, x>1$$

ASSIGNMENT:

Complete questions from NCERT Exercise 5.4 in your register

Link:

https://www.youtube.com/watch?v=XOTADOPp3As&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=9

Question 3:

Differentiate the function with respect to x.

$(\log x)^{\cos x}$

Solution 3:

Let $y=(\log x)^{\cos x}$

Taking logarithm or both the side, we obtain

$\log y=\cos x.\log(\log x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}.\frac{dy}{dx}=\frac{d}{dx}(\cos x)\text{x}\log(\log x)+\cos x\ \text{x}\frac{d}{dx}[\log(\log x)]$$

$$\Rightarrow\frac{1}{y}.\frac{dy}{dx}=-\sin x\log(\log x)+\cos x\ \text{x}\frac{1}{\text{logx}}.\frac{d}{dx}(\log x)$$

$$\Rightarrow\frac{dy}{dx}=y\left[-\sin x\log(\log x)+\frac{\cos x}{\log x}\ \text{x}\ \frac{1}{x}\right]$$

$$\therefore\frac{dy}{dx}=(\log x)^{\cos x}\left[\frac{\cos x}{x\log x}-\sin\text{x}\log(\log x)\right]$$

Question 5:

Differentiate the function with respect to x.

$(x+3)^2\cdot(x+4)^3\cdot(x+5)^4$

Solution 5:

Let

$$y=(x+3)^2\cdot(x+4)^3\cdot(x+5)^4$$

Taking logarithm on both sides, we obtain.

$\log y=\log(x+3)^2+\log(x+4)^3+\log(x+5)^4$

$\Rightarrow \log y=2\log(x+3)+3\log(x+4)+4\log(x+5)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{y}.\frac{dy}{dx}=2.\frac{1}{x+3}.\frac{d}{dx}(x+3)+3.\frac{1}{x+4}.\frac{d}{dx}(x+4)+4.\frac{1}{x+5}.\frac{d}{dx}(x+5)$

$\Rightarrow \frac{dy}{dx}=y\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$

$\Rightarrow \frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$

$\Rightarrow \frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$

$\Rightarrow \frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3\cdot\left[2(x^2+9x+20)+3(x^2+9x+15)+4(x^2+7x+12)\right]$

$\therefore \frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$

Question 7:

Differentiate the function with respect to x.

$(\log x)^x+x^{\log x}$

Solution 7:

Let $y=(\log x)^x+x^{\log x}$

Also, let $u=(\log x)^x$ and $v=x^{\log x}$

$\therefore y=u+v$

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$ ……..(1)

$u=(\log x)^x$

$\Rightarrow \log u=\log\left[(\log x)^x\right]$

$\Rightarrow \log u=x\log(\log x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}(x)x\log(\log x)+x.\frac{d}{dx}[\log(\log x)]$$

$$\Rightarrow \frac{du}{dx}=u\left[1x\log(\log x)+x.\frac{1}{\log x}.\frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\log(\log x)+\frac{x}{\log x}.\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\log(\log x)+\frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\frac{\log(\log x).\log x+1}{\log x}\right]$$

$$\frac{du}{dx}=(\log x)^{x-1}\left[1+\log x.\log(\log x)\right] \qquad \ldots\ldots(2)$$

$$v=x^{\log x}$$

$$\Rightarrow \log v=\log\left(x^{\log x}\right)$$

$$\Rightarrow \log v=\log x\log x=(\log x)^{2}$$

Differentiating both sides with respect to x , we obtain

$$\frac{1}{v}.\frac{dv}{dx}=\frac{d}{dx}\left[(\log x)^{2}\right]$$

$$\Rightarrow \frac{1}{v}.\frac{dv}{dx}=2(\log x).\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dv}{dx}=2x^{\log x}\frac{\log x}{x}$$

$$\Rightarrow \frac{dv}{dx}=2x^{\log x-1}.\log x \qquad \ldots.(3)$$

Therefore, from (1),(2), and (3), we obtain

$$\frac{dy}{dx}=(\log x)^{x-1}[1+\log x.\log(\log x)]+2x^{\log x-1}\cdot\log x$$

Question 10:

Differentiate the function with respect to x.

$$x^{x\cos x}+\frac{x^{2}+1}{x^{2}-1}$$

Solution 10:

Let $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Also, let $u = x^{x\cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$\therefore y = u + v$

$u = x^{x\cos x}$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x).\cos x \log x + x.\frac{d}{dx}(\cos x).\log x + x\cos x.\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1.\cos x.\log x + x.(-\sin x)\log x + x\cos x.\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}(cox\log x - x\sin x\log x + \cos x)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1+\log x) - x\sin x\log x\right] \qquad \text{....(2)}$$

$$v = \frac{x^2+1}{x^2-1}$$

$$\Rightarrow \log v = \log(x^2+1) - \log(x^2-1)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v} = \frac{dv}{dx} = \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{2x(x^2-1) - 2x(x^2+1)}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{x^2+1}{x^2-1} \text{ x} \left[\frac{-4x}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{(x^2-1)^2} \qquad \text{.......(3)}$$

Therefore, from (1), (2) and (3), we obtain

ASSIGNMENT:

Complete questions from NCERT Exercise 5.5 in your register

Link:

https://www.youtube.com/watch?v=xTd73BOsaH4&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=10

DAY-4

LESSON DEVELOPMENT

Differentiation of Functions in Parametric Form: A relation expressed between two variables x and y in the form x = f(t), y = g(t) is said to be parametric form with t as a parameter, when

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

(whenever $\frac{dx}{dt} \neq 0$)

Note: dy/dx is expressed in terms of parameter only without directly involving the main variables x and y.

Question 2:

If x and y are connected parametrically by the equation, without eliminating the parameter, find

$\frac{dy}{dx}$

$x = a\cos\theta,\ y = b\cos\theta$

Solution 2:

The given equations are $x = a\cos\theta$ and $y = b\cos\theta$

Then, $\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = a(-\sin\theta) = -a\sin\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\cos\theta) = b(-\sin\theta) = -b\sin\theta$

$$\therefore \frac{dy}{dx} \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$$

Question 4:

If x and y are connected parametrically by the equation, without eliminating the parameter, find

$$\frac{dy}{dx}$$

$$x = 4t,\ y = \frac{4}{t}$$

Solution 4:

The equations are $x = 4t$ and $y = \frac{4}{t}$

$$\frac{dx}{dt} = \frac{d}{dt}(4t) = 4$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = 4\cdot\frac{d}{dt}\left(\frac{1}{t}\right) = 4\cdot\left(\frac{-1}{t^2}\right) = \frac{-4}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{-4}{t^2}\right)}{4} = \frac{-1}{t^2}$$

Question 8:

If x and y are connected parametrically by the equation, without eliminating the parameter, find

$$\frac{dy}{dx}$$

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right),\ y = a\sin t$$

The given equations are $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$ and $y = a\sin t$

Then, $\frac{dx}{dt} = a\cdot\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\left(\log\tan\frac{t}{2}\right)\right]$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}\cdot\sec^2\frac{t}{2}\cdot\frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = a\frac{d}{dt}(\sin t) = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a\cos t}{\left(a\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

Question 11:

If $x = \sqrt{a^{\sin -1 t}}$, $y = \sqrt{a^{\cos -1 t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

Solution 11:

The given equations are $x = \sqrt{a^{\sin -1 t}}$ and $y = \sqrt{a^{\cos -1 t}}$

$x = \sqrt{a^{\sin -1 t}}$ and $y = \sqrt{a^{\cos -1 t}}$

$\Rightarrow x = \left(a^{\sin -1 t}\right)$ and $y = \left(a^{\cos -1 t}\right)^{\frac{1}{2}}$

$\Rightarrow x = a^{\frac{1}{2}\sin -1 t}$ and $y = a^{\frac{1}{2}\cos -1 t}$

Consider $x = a^{\frac{1}{2}\sin -1 t}$

Taking logarithm on both sides, we obtain.

$$\log x = \frac{1}{2}\sin^{-1} t \log a$$

$$\therefore \frac{1}{x}\cdot\frac{dx}{dt} = \frac{1}{2}\log a \cdot \frac{d}{dt}(\sin^{-1} t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{x}{2}\log a \cdot \frac{1}{\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{x\log a}{2\sqrt{1-t^2}}$$

Then, consider

$$y = a^{\frac{1}{2}\cos^{-1} t}$$

Taking logarithm on both sides, we obtain.

$$\log y = \frac{1}{2}\cos^{-1} t \log a$$

$$\therefore \frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{2}\log a\cdot\frac{d}{dt}\left(\cos^{-1}t\right)$$

$$\Rightarrow \frac{dy}{dt}=\frac{y\log a}{2}\cdot\left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt}=\frac{-y\log a}{2\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{\left(\frac{-y\log a}{2\sqrt{1-t^2}}\right)}{\left(\frac{x\log a}{2\sqrt{1-t^2}}\right)}=-\frac{y}{x}$$

Hence proved.

ASSIGNMENT:

Complete questions from NCERT Exercise 5.6 in your register

Link:

https://www.youtube.com/watch?v=nQZ1HY-AMAg&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=11

Second order Derivative: It is the derivative of the first order derivative.

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

Link:

https://www.youtube.com/watch?v=QVPSM2gBXc4&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=5

Question 3:

Find the second order derivatives of the function. $x \cdot \cos x$

Solution 3:

Let $y = x \cdot \cos x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x \cdot \cos x) = \cos x \cdot \frac{d}{dx}(x) + x\frac{d}{dx}(\cos x) = \cos x \cdot 1 + x(-\sin x) = \cos x - x\sin x$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[\cos x - \sin x] = \frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x)$$

$$= -\sin x - \left[\sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)\right]$$

$$= -\sin x - (\sin x + \cos x)$$

$$= -(x\cos x + 2\sin x)$$

Question 5:

Find the second order derivatives of the function. $x^3 \log x$

Solution 5:

Let $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 \log x\right] = \log x \cdot \frac{d}{dx}(x^3) + x^3 \cdot \frac{d}{dx}(\log x)$$

$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2$$

$$= x^2(1 + 3\log x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[x^2(1 + 3\log x)\right]$$

$$= (1 + 3\log x) \cdot \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(1 + 3\log x)$$

$$= (1 + 3\log x) \cdot 2x + x^3 \cdot \frac{3}{x}$$

$$= 2x + 6\log x + 3x$$

$$= 5x + 6x\log x$$

$$= x(5 + 6\log x)$$

Question 7:
Find the second order derivatives of the function. $e^{6x}\cos 3x$

Solution 7:
Let $y = e^{6x}\cos 3x$
Then,

$$\frac{dy}{dx} = \frac{d}{dx}(e^{6x}\cos 3x) = \cos 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\cos 3x)$$

$$= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x)$$

$$= 6e^{6x}\cos 3x - 3e^{6x}\sin 3x \;\ldots\ldots(1)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(6e^{6x}\cos 3x - 3e^{6x}\sin 3x) = 6 \cdot \frac{d}{dx}(e^{6x}\cos 3x) - 3 \cdot \frac{d}{dx}(e^{6x}\sin 3x)$$

$$= 6 \cdot \left[6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\sin 3x)\right] \qquad \text{[using (1)]}$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 3\left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x - 3\right]$$

$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x - 9e^{6x}\cos 3x$$

$$= 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$

$$= 9e^{6x}(3\cos 3x - 4\sin 3x)$$

Question 9:
Find the second order derivatives of the function. $\log(\log x)$

Solution 9:
Let $y = \log(\log x)$
Then,

$$\frac{dy}{dx} = \frac{d}{dx}[\log(\log x)] = \frac{1}{\log x} \cdot \frac{d}{dx}(\log x) = \frac{1}{\log x} = (x\log x)^{-1}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[(x\log x)^{-1}\right] = (-1) \cdot (x\log x)^{-2}\frac{d}{dx}(x\log x)$$

$$= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log x)\right]$$

$$= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot 1x \cdot \frac{1}{x}\right] = \frac{-1(1+\log x)}{(x\log x)^2}$$

Question 11:

If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

Solution 11:

It is given that, $y = 5\cos x - 3\sin x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(5\cos x) - \frac{d}{dx}(3\sin x) = 5\frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$= 5(-\sin x) - 3\cos x = -(5\sin x + 3\cos x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[-(5\sin x + 3\cos x)]$$

$$= -\left[5 \cdot \frac{d}{dx}(\sin x) + 3 \cdot \frac{d}{dx}(\cos x)\right]$$

$$= [5\cos x + 3(-\sin x)]$$

$$= -[5\cos x - 3\sin x]$$

$$= -y$$

$$\therefore \frac{d^2y}{dx^2} + y = 0$$

Hence, proved.

Question 14:

If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Solution 14:

It is given that, $y = Ae^{mx} + Be^{nx}$

Then,

$$\frac{dy}{dx} = A \cdot \frac{d}{dx}(e^{mx}) + B \cdot \frac{d}{dx}(e^{nx}) = A \cdot e^{mx} \cdot \frac{d}{dx}(mx) + B \cdot e^{nx} \cdot \frac{d}{dx}(nx) = Ame^{mx} + Bne^{nx}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(Ame^{mx} + Bne^{nx}\right) = Am \cdot \frac{d}{dx}(e^{mx}) + Bn \cdot \frac{d}{dx}(e^{nx})$$

$$= Am \cdot e^{mx} \cdot \frac{d}{dx}(mx) + bn \cdot e^{nx} \cdot \frac{d}{dx}(nx) = Am^2e^{mx} + Bn^2e^{nx}$$

$$\therefore \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= Am^2e^{mx} + Bn^2e^{nx} - (m+n) \cdot (Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$$

$$= Am^2ex^{mx} + Bn^2e^{nx} - Am^2ex^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{mx} + Amne^{mx} + Bmne^{nx}$$

$$= 0$$

Hence, Proved.

ASSIGNMENT:

Complete questions from NCERT Exercise 5.7 in your register

Link:

https://www.youtube.com/watch?v=l_d3I3DxhGo&list=PLKKfKV1b9e8rueJyU6AX8Vk1M0kDNDMYa&index=12

Some Standard Derivatives

(i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(iv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(v) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(vi) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(vii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(viii) $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

(ix) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(x) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(xi) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

(xii) $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$

(xiii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(xiv) $\frac{d}{dx}(\text{constant}) = 0$

(xv) $\frac{d}{dx}(e^x) = e^x$

(xvi) $\frac{d}{dx}(\log_e x) = \frac{1}{x}, x > 0$

(xvii) $\frac{d}{dx}(a^x) = a^x \log_e a, a > 0$

Some Useful Substitutions for Finding Derivatives Expression

Expression	Substitution
(i) $a^2 + x^2$	$x = a\tan\theta$ or $x = a\cot\theta$
(ii) $a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$
(iii) $x^2 - a^2$	$x = a\sec\theta$ or $x = a\,\text{cosec}\theta$
(iv) $\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$
(v) $\sqrt{\frac{a^2-x^2}{a^2+x^2}}$ or $\sqrt{\frac{a^2+x^2}{a^2-x^2}}$	$x^2 = a^2\cos 2\theta$

EXTRA QUESTIONS:

Q1)

The greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x =$ __________ .

1 mark (CBSE 2020)

Q2)

If $x = at^2$, $y = 2at$, then find $\frac{d^2y}{dx^2}$.

2 marks (CBSE 2020)

Q3)

If $y = e^{x^2 \cos x} + (\cos x)^x$, then find $\frac{dy}{dx}$.

4 marks (CBSE 2020)

Q4)

If $f(x) = x + 1$, find $\frac{d}{dx}(fof)(x)$.

1 mark (CBSE 2019)

Q5)

If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

4 marks (CBSE 2019)

Q6)

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

4 marks (CBSE 2019)

Q7)

Differentiate $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ with respect to x.

2 marks (CBSE 2018)

Q8)

MAPS

If $\sin y = x \cos(a + y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$.

Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

4 marks (CBSE 2018)

Q9)

If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

4 marks (CBSE 2018)

Q10)

If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2) \frac{d^2y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0$.

4 marks (CBSE 2018)

Q11)

Determine the value of 'k' for which the following function is continuous at x = 3 :

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x - 3} & , \quad x \neq 3 \\ k & , \quad x = 3 \end{cases}$$

1 mark (CBSE 2017)

Q12)

If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

4 marks (CBSE 2017)

Q13)

If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

4 marks (CBSE 2017)

Q14)

Find the value(s) of k so that the following function is continuous at $x = 0$

$$f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x} & if\ x \neq 0 \\ \frac{1}{2} & if\ x = 0 \end{cases}$$

2 marks Board SQP 2020-21

Q15)

If y = $e^{x \sin^2 x}$ + $(\sin x)^x$, find $\frac{dy}{dx}$.

3 marks Board SQP 2020-21

Q16)

Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 2$ is not differentiable at $x = 1$

3 marks Board SQP 2020-21

Q17)

If $x = a\ \sec\theta\ , y = b\ tan\theta\ find\ \frac{d^2y}{dx^2}\ at\ x = \frac{\pi}{6}$

3 marks Board SQP 2020-21

Q18)

If x = a sin 2t (1 + cos 2t) and y = b cos 2t (1 − cos 2t), find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$.

4 marks (CBSE 2016)

Q19)

If y = x^x prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$. 4 marks (CBSE 2016)

Q20)

Find the values of p ans q for which [4]

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & , & \text{if } x < \frac{\pi}{2} \\ p & , & \text{if } x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi - 2x)^2} & , & \text{if } x > \frac{\pi}{2} \end{cases}$$

is continuous at $x = \pi/2$.

4 marks (CBSE 2016)