Mount Abu Public School

H-Block, Sector-18, Rohini, New Delhi-110085

Subject - Mathematics
Class - IX

Planner

Chapter- Number Systems

INTRODUCTION

Introduction to Number Systems

Numbers

Number: Arithmetical value representing a particular quantity. The various types of numbers are Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, Real Numbers etc.

Natural Numbers

Natural numbers**(N)** are positive numbers i.e. 1, 2, 3 ..and so on.

Whole Numbers

Whole numbers **(W)** are 0, 1, 2,..and so on. Whole numbers are all Natural Numbers including '0'. Whole numbers do not include any fractions, negative numbers or decimals.

Integers

Integers are the numbers that includes whole numbers along with the negative numbers.

Rational Numbers

A number 'r' is called a rational number if it can be written in the form p/q, where p and q are integers and $q \neq 0$.

Irrational Numbers

Any number that cannot be expressed in the form of p/q, where p and q are integers and $q\neq0$, is an irrational number. Examples: $\sqrt{2}$, 1.010024563…, e, π

Real Numbers

Any number which can be represented on the number line is a Real Number**(R).** It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

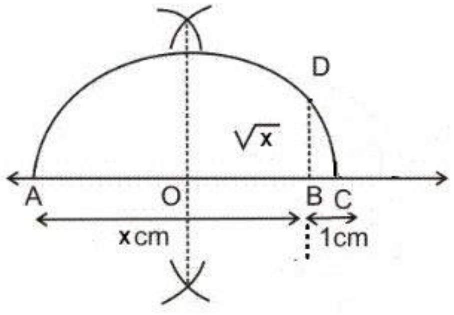
Irrational Numbers

Representation of Irrational numbers on the Number line

Let $\sqrt{x}$ be an irrational number. To represent it on the number line we will follow the following steps:

- Take any point A. Draw a line AB = x units.
- Extend AB to point C such that BC = 1 unit.
- Find out the mid-point of AC and name it 'O'. With 'O' as the centre draw a semi-circle with radius OC.
- Draw a straight line from B which is perpendicular to AC, such that it intersects the semi-circle at point D.

Length of BD=√x.



Constructions to Find the root of *x*.

- With BD as the radius and origin as the centre, cut the positive side of the number line to get √x.

Identities for Irrational Numbers

Arithmetic operations between:

- rational and irrational will give an irrational number.
- irrational and irrational will give a rational or irrational number.

Example : 2 × √3 = 2√3 i.e. irrational. √3 × √3 = 3 which is rational.

Identities for irrational numbers

If a and b are real numbers then:

-
- √ab = √a√b
- √ab = √a√b
- (√a+√b) (√a-√b) = a – b
- (a+√b)(a−√b) = a^2−b
- (√a+√b)(√c+√d) = √ac+√ad+√bc+√bd
- (√a+√b)(√c−√d) = √ac−√ad+√bc−√bd
- $(\sqrt{a}+\sqrt{b})^2$ = a+2√(ab)+b

Rationalisation

Rationalisation is converting an irrational number into a rational number. Suppose if we have to rationalise 1/√a.
1/√a × 1/√a = 1/a

Rationalisation of 1/√a+b:

(1/√a+b) × (1/√a−b) = (1/a−b^2)

Laws of Exponents for Real Numbers

If a, b, m and n are real numbers then:

- $a^m \times a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^m/a^n = a^{m-n}$
- $a^m b^m = (ab)^m$

Here, *a* and *b* are the bases and *m* and *n* are exponents.

Exponential representation of irrational numbers

If a > 0 and n is a positive integer, then: n√a=a1n Let a > 0 be a real number and p and q be rational numbers, then:

- $a^p \times a^q = a^{p+q}$
- $(a^p)^q = a^{pq}$
- $a^p/a^q = a^{p-q}$
- $a^p b^p = (ab)^p$

Decimal Representation of Rational Numbers

Decimal expansion of Rational and Irrational Numbers

The decimal expansion of a rational number is either terminating or non- terminating and recurring.

Example: 1/2 = 0.5 , 1/3 = 3.33…….
The decimal expansion of an irrational number is non terminating and non-recurring.
Examples: √2 = 1.41421356..

Expressing Decimals as rational numbers

Case 1 – Terminating Decimals

Example – 0.625

Let x=0.625

If the number of digits after the decimal point is y, then multiply and divide the number by 10^y.

So, x = 0.625 × 1000/1000 = 625/1000 Then, reduce the obtained fraction to its simplest form.

Hence, x = 5/8

Case 2: Recurring Decimals

If the number is non-terminating and recurring, then we will follow the following steps to convert it into a rational number:

Example – $1.0\overline{42}$

Step 1. Let x = $1.0\overline{42}$ (1)

Step 2. Multiply the first equation with 10^y, where y is the number of digits that are recurring.

- Thus, 100x = $104.2\overline{42}$ (2)**Steps 3.** Subtract equation 1 from equation 2.On subtracting equation 1 from 2, we get99x = 103.2x=103.2/99 = 1032/990

  Which is the required rational number.

  Reduce the obtained rational number to its simplest form Thus,

  x=172/165

Assignment:

1. Find five rational numbers between 3/5 and 4/5.
2. Locate $\sqrt{2}$ on the number line.
3. State whether the following statements are true or false. Justify your answers.
   (i)Every irrational number is a real number.
   (ii) Every point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.
   (iii) Every real number is an irrational number.
4. Show that 3.142678 is a rational number. In other words, express 3.142678 in the form p/q, where *p* and *q* are integers and *q* ¹ 0.
5. Find an irrational number between 1/7 and 2/7.
6. Show how √5 can be represented on the number line.
7. Express the following in the form p/q, where p and q are integers and q ≠ 0..

   (i) $\mathbf{0.\overline{6}}$ (ii) $0.4\overline{7}$
8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.
9. Simplify each of the following expressions:

   (i)(3+√3)(2+√2)

   (ii) $(\sqrt{5}+\sqrt{2})^2$
10. Rationalise the denominator of $\dfrac{5}{\sqrt{3}-\sqrt{5}}$.
11. Find: (i)$64^{1/2}$

    (ii)$32^{1/5}$
12. : Simplify (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}$

Chapter- Polynomials

INTRODUCTION

Polynomials are expressions with one or more terms with a non-zero coefficient. A polynomial can have more than one term. In the polynomial, each expression in it is called a **term**. Suppose $x^2 + 5x + 2$ is polynomial, then the expressions x^2, 5x, and 2 are the terms of the polynomial. Each term of the polynomial has a **coefficient**. For example, if 2x + 1 is the polynomial, then the coefficient of x is 2.

The real numbers can also be expressed as polynomials. Like 3, 6, 7, are also polynomials without any variables. These are called **constant polynomials**. The constant polynomial 0 is called **zero polynomial**. The exponent of the polynomial should be a whole number. For example, $x^{-2} + 5x + 2$, cannot be considered as a polynomial, since the exponent of x is -2, which is not a whole number.

The highest power of the polynomial is called the **degree of the polynomial**. For example, in $x^3 + y^3 + 3xy(x + y)$, the degree of the polynomial is 3. For a non zero constant polynomial, the degree is zero. Apart from these, there are other types of polynomials such as:

- Linear polynomial – of degree one
- Quadratic Polynomial- of degree two
- Cubic Polynomial – of degree three

Example of polynomials are:

- 20
- x + y
- 7a + b + 8
- w + x + y + z
- $x^2 + x + 1$

Polynomials in One Variable

Polynomials in one variable are the expressions which consist of only one type of variable in the entire expression.

Example of polynomials in one variable:

- 3a
- $2x^2 + 5x + 15$

Some important points in Polynomials are given below:

- An algebraic expression $p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots a_n$ is a polynomial where a_0, a_1, ………. a_n are real numbers and n is non-negative integer.
- A term is either a variable or a single number or it can be a combination of variable and numbers.
- The degree of the polynomial is the highest power of the variable in a polynomial.
- A polynomial of degree 1 is called as a **linear polynomial**.
- A polynomial of degree 2 is called a **quadratic polynomial**.
- A polynomial of degree 3 is called a **cubic polynomial**.
- A polynomial of 1 term is called a **monomial**.
- A polynomial of 2 terms is called **binomial**.
- A polynomial of 3 terms is called a **trinomial**.
- A real number ‘a’ is a zero of a polynomial p(x) if p(a) = 0, where a is also known as root of the equation p(x) = 0.

- A linear polynomial in one variable has a unique zero, a polynomial of a non-zero constant has no zero, and each real number is a zero of the zero polynomial.
- **Remainder Theorem:** If p(x) is any polynomial having degree greater than or equal to 1 and if it is divided by the linear polynomial x – a, then the remainder is p(a).
- **Factor Theorem** : x – c is a factor of the polynomial p(x), if p(c) = 0. Also, if x – c is a factor of p(x), then p(c) = 0.
- The degree of the zero polynomial is not defined.
- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
- $(x – y)^3 = x^3 – y^3 – 3xy(x – y)$
- $x^3 + y^3 + z^3 – 3xyz = (x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx)$

Examples

Example 1:

Write the coefficients of x in each of the following:

- $3x + 1$
- $23x^2 – 5x + 1$

Solution:

In $3x + 1$, the coefficient of x is 3.

In $23x^2 – 5x + 1$, the coefficient of x is -5.

Example 2:

What are the degrees of following polynomials?

1. $3a^2 + a – 1$
2. $32x^3 + x – 1$

Solution:

1. $3a^2 + a – 1$: The degree is 2
2. $32x^3 + x – 1$: The degree is 3

Assignment:

1. Find value of polynomial $2x^2 + 5x + 1$ at $x = 3$.
2. Check whether $x = -1/6$ is zero of the polynomial $p(a) = 6a + 1$.
3. Divide $3a^2 + x – 1$ by $a + 1$.
4. Factorise each of the following:
   - $4x^2 + 9y^2 + 16z^2 + 12xy – 24yx – 16xz$
   - $2x^2 + y^2 + 8z^2 – 2\sqrt{2}xy + 4\sqrt{2}yz – 8xz$
5. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

   (i) $4x^2–3x+7$

   (ii) $y^2+\sqrt{2}$

   (iii) $3\sqrt{t}+t\sqrt{2}$
6. Write the coefficients of x^2 in each of the following:

   (i) $2+x^2+x$

(ii) $2x-1$

7. Write the degree of each of the following polynomials:

(i) $5x^3+4x^2+7x$

(ii) $4–y^2$

(iii) $5t–\sqrt{7}$

8. Classify the following as linear, quadratic and cubic polynomials:

(i) x^2+x

(ii) $x–x^3$

(iii) $y+y^2+4$

9. Find the value of each of the following polynomials at the indicated value of variables:

(i) $p(x) = 5x^2 – 3x + 7$ at $x = 1$.
(ii) $q(y) = 3y^3 – 4y + 11$ at $y = 2$.
(iii) $p(t) = 4t^4 + 5t^3 – t^2 + 6$ at $t = a$.

10. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 5x – p$, $x = 4/5$
(ii) $p(x) = x^2$, $x = 0$
(iii) $p(x) =2x+1$, $x = ½$

11. Find the zero of the polynomial in each of the following cases:
(i) $p(x) = x–5$
(ii) $p(x) = cx+d$, $c \neq 0$, c, d are real numbers.

12. Find the remainder obtained on dividing $p(x) = x^3 + 1$ by $x + 1$.
13. Find the remainder when $x^3−ax^2+6x−a$ is divided by $x-a$.
14. Use suitable identities to find the following products:
$(x+8)(x –10)$

15. Factorize the following using appropriate identities: $9x^2+6xy+y^2$
16. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x−y+z)^2$

(iii) $(−2x+3y+2z)^2$

(iv) $(3a –7b–c)^2$

17. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

18. Factorise each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$

(ii) $8a^3–b^3–12a^2b+6ab^2$

(iii) $27–125a^3–135a +225a^2$

Chapter- Coordinate Geometry

INTRODUCTION

Cartesian System

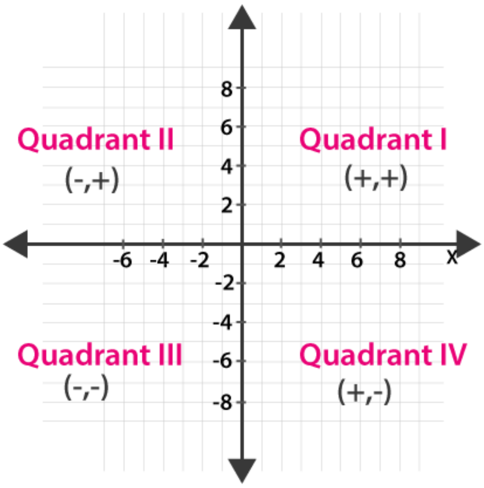
Cartesian plane & Coordinate Axes

Cartesian Plane: A cartesian plane is defined by **two perpendicular number lines**, A horizontal line(x−axis) and a vertical line (y−axis).
These lines are called coordinate axes. The Cartesian plane extends infinitely in all directions.

Origin: The coordinate axes intersect each other at right angles, **The point of intersection** of these two axes is called Origin.

Quadrants

The cartesian plane is divided into four equal parts, called **quadrants.** These are named in the order as I, II, III and IV starting with the upper right and going around in anticlockwise direction.



Points in different Quadrants.

Signs of coordinates of points in different quadrants:

I Quadrant: '+' x – coordinate and '+' y – coordinate. E.g. (2, 3)

II Quadrant: '-' x – coordinate and '+' y – coordinate. E.g. (-1, 4)

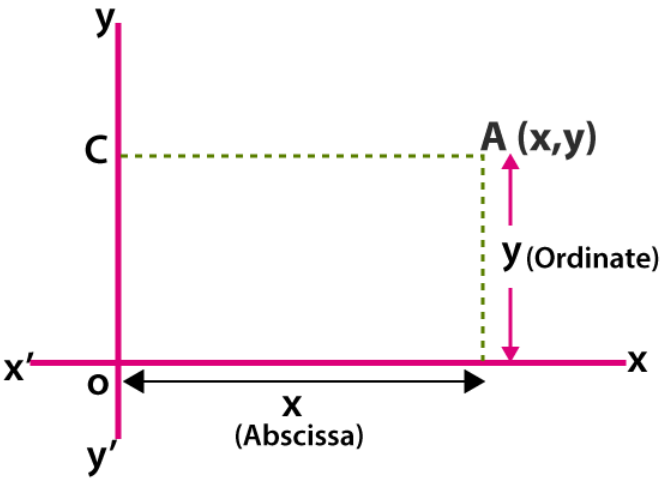
III Quadrant: '-' x – coordinate and '-' y – coordinate. E.g. (-3, -5)

IV Quadrant: '+' x – coordinate and '-' y – coordinate. E.g. (6, -1)

Plotting on a Graph

Representation of a point on the Cartesian plane

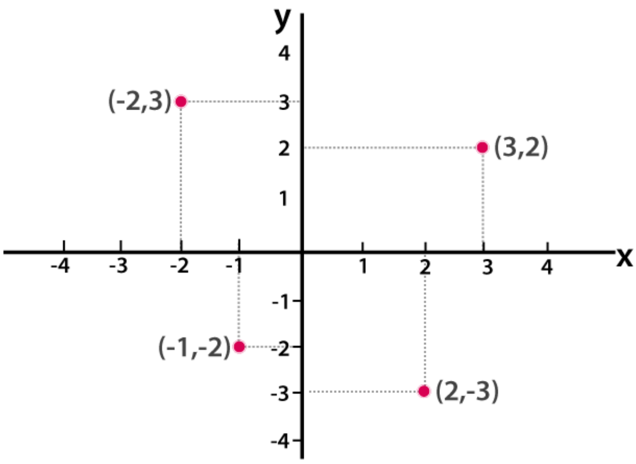
Using the co-ordinate axes, we can describe any point in the plane using an ordered pair of numbers. A point **A** is represented by an ordered pair (x, y) where x is the **abscissa** and y is the **ordinate** of the point.



Position of a point in a plane

Plotting a point

The coordinate points will define the location in the cartesian plane. The first point (x) in the coordinates represents the horizontal axis, and the second point in the coordinates (y) represents the vertical axis.
Consider an example, Point (3, 2) is 3 units away from the positive y-axis and 2 units away from the positive x-axis. Therefore, point (3, 2) can be plotted, as shown below. Similarly, (-2, 3), (-1, -2) and (2, -3) are plotted.



Plotting a point in the plane

Assignment:

1. Write the answer of each of the following questions:
   (i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?
   (ii) What is the name of each part of the plane formed by these two lines?
   (iii) Write the name of the point where these two lines intersect.
2. Locate the points (5, 0), (0, 5), (2, 5), (5, 2), (–3, 5), (–3, –5), (5, –3) and (6, 1) in the Cartesian plane.
3. In which quadrant or on which axis do each of the points (– 2, 4), (3, – 1), (– 1, 0),

(1, 2) and (– 3, – 5) lie? Verify your answer by locating them on the Cartesian plane.

4. Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	-2	-1	0	1	3
y	8	7	-1.25	3	-1

5. See Fig, and write the following:

i. The coordinates of B.

ii. The coordinates of C.

iii. The point identified by the coordinates (–3, –5).

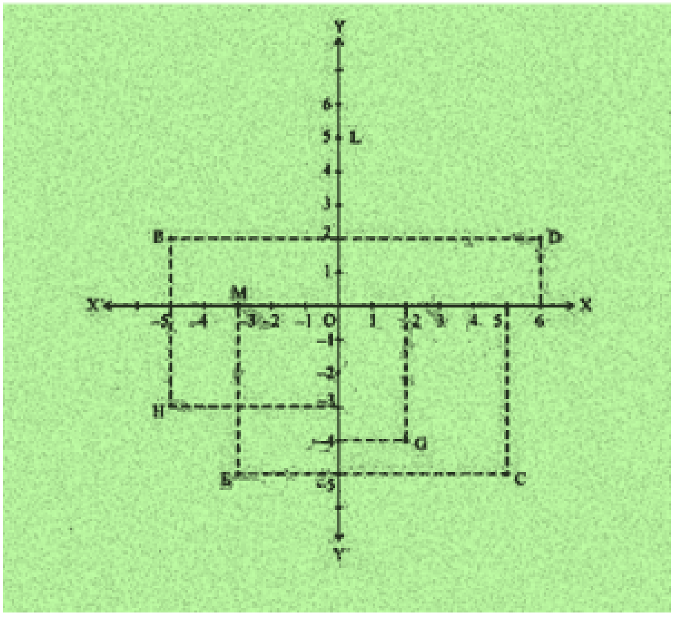
iv. The point identified by the coordinates (2, – 4).

v. The abscissa of the point D.

vi. The ordinate of the point H.

vii. The coordinates of the point L.

viii. The coordinates of the point M.



6. Plot the following ordered pairs (x, y) of numbers as points in the Cartesian plane. Use the scale 1cm = 1 unit on the axes.

x	– 3	0	– 1	4	2
y	7	–3.5	– 3	4	– 3

Chapter- Linear Equations In Two Variables

INTRODUCTION

Linear equation in one variable

When an equation has only one variable of degree one, then that equation is known as linear equation in one variable.

- Standard form: ax+b=0, where a and b ϵ R & a ≠ 0
- Examples of linear equation in one variable are :

– 3x-9 = 0
– 2t = 5

Linear equation in 2 variables

When an equation has two variables both of degree one, then that equation is known as linear equation in two variables.
Standard form: ax+by+c=0, where a, b, c ϵ R & a, b ≠ 0
Examples of linear equations in two variables are:
– 7x+y=8
– 6p-4q+12=0

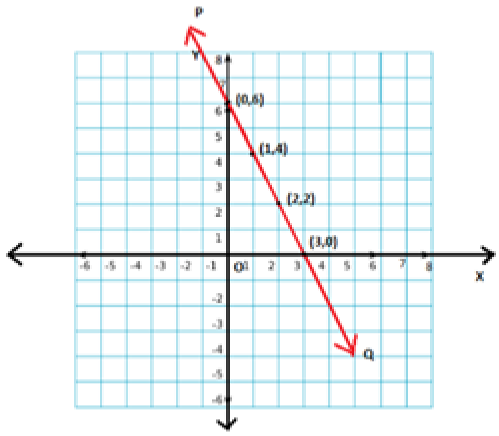
The solution of linear equation in 2 variables

A linear equation in two variables has a pair of numbers that can satisfy the equation. This pair of numbers is called as the solution of the linear equation in two variables.

- The solution can be found by assuming the value of one of the variable and then proceed to find the other solution.
- There are infinitely many solutions for a single linear equation in two variables.

Graphical representation of a linear equation in 2 variables

- Any linear equation in the standard form ax+by+c=0 has a pair of solutions in the form (x,y), that can be represented in the coordinate plane.
- When an equation is represented graphically, it is a straight line that may or may not cut the coordinate axes.



Solutions of Linear equation in 2 variables on a graph

- A linear equation ax+by+c=0 is represented graphically as a straight line.
- Every point on the line is a solution for the linear equation.
- Every solution of the linear equation is a point on the line.

Lines passing through the origin

- Certain linear equations exist such that their solution is (0, 0). Such equations when represented graphically pass through the origin.
- The coordinate axes namely x-axis and y-axis can be represented as y=0 and x=0, respectively.

Lines parallel to coordinate axes

- Linear equations of the form y=a, when represented graphically are lines parallel to the x-axis and a is the y-coordinate of the points in that line.
- Linear equations of the form x=a, when represented graphically are lines parallel to the y-axis and a is the x-coordinate of the points in that line.

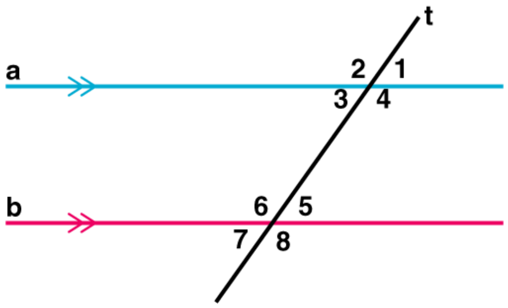
Assignment:

1. Write each of the following as an equation in two variables:
   (i) $x = -5$ (ii) $y = 2$ (iii) $2x = 3$ (iv) $5y = 2$
2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:
   (i) $2x + 3y = 9.35$ (ii) $x - y/5 - 10 = 0$ (iii) $-2x + 3y = 6$ (iv) $x = 3y$
   (v) $2x = -5y$ (vi) $3x + 2 = 0$
3. Find four different solutions of the equation $x + 2y = 6$.
4. Which one of the following options is true, and why?
   $y = 3x + 5$ has
   (i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions
5. Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:
   (i) (0, 2) (ii) (2, 0) (iii) (4, 0) (iv) $\left(2 , 4\,2 \right)$ (v) (1, 1)
6. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.
7. Draw the graph of each of the following linear equations in two variables:
   (i) $x + y = 4$ (ii) $x - y = 2$ (iii) $y = 3x$ (iv) $3 = 2x + y$
8. Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?
9. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.
10. Yamini and Fatima, two students of Class IX of a school, together contributed ` 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as ` x and y.) Draw the graph of the same.
11. Solve the equation $2x + 1 = x - 3$, and represent the solutions on (i) the number line (ii) the Cartesian plane.
12. Give the geometric representations of $y = 3$ as an equation
    (i) in one variable
    (ii) in two variables
13. Give the geometric representations of $2x + 9 = 0$ as an equation
    (i) in one variable
    (ii) in two variables

Chapter- Lines and Angles

INTRODUCTION

Parallel lines with a transversal



- $\angle 1=\angle 5, \angle 2=\angle 6, \angle 4=\angle 8$ and $\angle 3=\angle 7$(Corresponding angles)
- $\angle 3=\angle 5, \angle 4=\angle 6$ (Alternate interior angles)
- $\angle 1=\angle 7, \angle 2=\angle 8$ (Alternate exterior angles)

Lines parallel to the same line

Lines that are parallel to the same line are also parallel to each other.

Angles and types of angles

When 2 rays originate from the same point at different directions, they form an angle.
– The rays are called arms and the common point is called the vertex

– Types of angles : (i) Acute angle $0^\circ<a<90^\circ$

(ii) Right angle $a=90^\circ$

(iii) Obtuse angle : $90^\circ<a<180^\circ$

(iv) Straight angle $=180^\circ$

(v) Reflex Angle $180^\circ<a<360^\circ$

(vi) Angles that add up to 90° are complementary angles

(vii) Angles that add up to 180° are called supplementary angles.

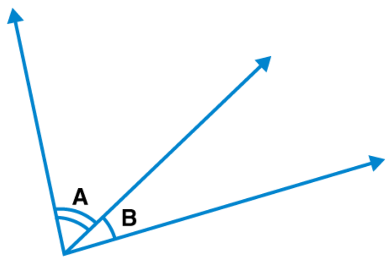
To know more about Angles and Types of Angles, visit here.

Intersecting and Non-Intersecting lines

- When 2 lines meet at a point they are called intersecting
- When 2 lines never meet at a point, they are called non-intersecting or parallel lines

Adjacent angles

2 angles are adjacent if they have the same vertex and one common point.



Linear Pair

When 2 adjacent angles are supplementary, i.e they form a straight line (add up to 180°), they are called a linear pair.

Vertically opposite angles

When two lines intersect at a point, they form equal angles that are vertically opposite to each other.

To know more about Lines and Angles, visit here.

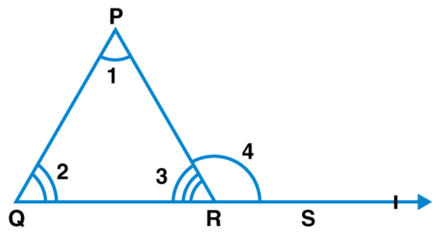
Basic Properties of a Triangle

Triangle and sum of its internal angles

Sum of all angles of a triangle add up to 180°

An exterior angle of a triangle = sum of opposite internal angles

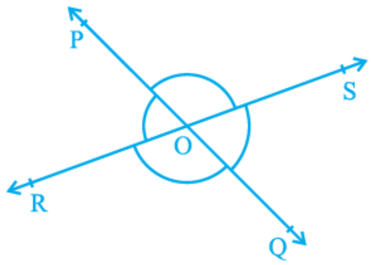
– If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles



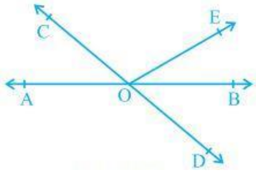
– $\angle 4 = \angle 1 + \angle 2$

Assignment:

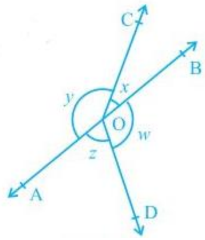
1. In Fig. lines PQ and RS intersect each other at point O. IfÐ POR : Ð ROQ = 5 : 7, find all the angles.



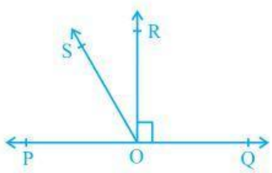
2. In Fig., lines AB and CD intersect at O. If AOC +BOE = 70° and BOD = 40°, find BOE and reflex COE.



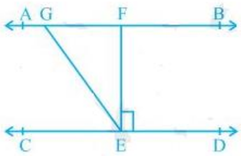
3. In Fig,PQR = PRQ, then prove that PQS = PRT



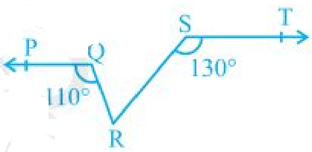
4. In Fig, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ROS = ½ (QOS – POS).



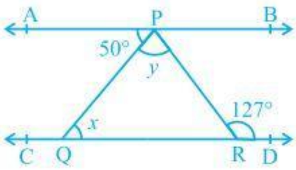
5. It is given that XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ZYP, find XYQ and reflex QYP.
6. In Fig. ,if AB CD, EF ⊥ CD and GED = 126°, find AGE, GEF and FGE.



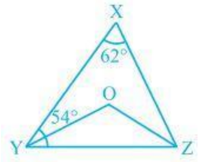
7. If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.
8. In Fig. , if PQ ST, PQR = 110° and RST = 130°, find QRS.



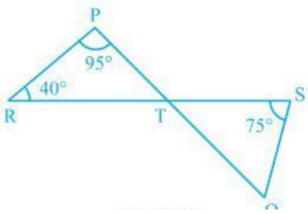
9. In Fig. , if AB CD, APQ = 50° and PRD = 127°, find x and y.



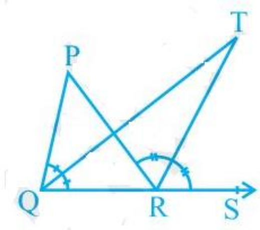
10. In Fig., X = 62°, XYZ = 54°. If YO and ZO are the bisectors of XYZ and XZY respectively of Δ XYZ, find OZY and YOZ.



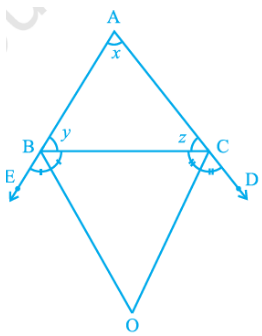
11. In Fig., if lines PQ and RS intersect at point T, such that PRT = 40°, RPT = 95° and TSQ = 75°, find SQT.



12. In Fig., the side QR of ΔPQR is produced to a point S. If the bisectors of PQR and PRS meet at point T, then prove that QTR = ½ QPR.



13. In Fig., the sides AB and AC of DABC are produced to points E and D respectively.If bisectors BO and CO of Ð CBE and Ð BCD respectively meet at point O, then prove that Ð BOC = 90° – $\frac{1}{2}$ ÐBAC.

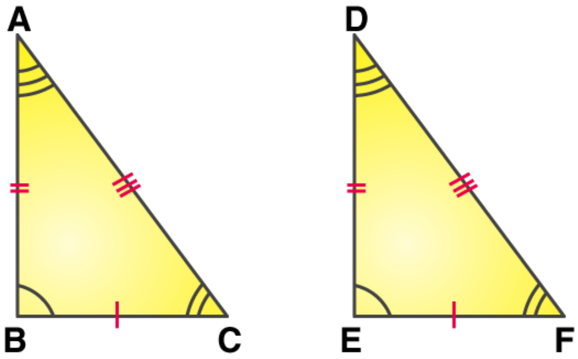


Chapter- Triangles

INTRODUCTION

Congruent Triangles

In a pair of triangles if all three corresponding sides and three corresponding angles are exactly equal, then the triangles are said to be congruent.



In congruent triangles, the corresponding parts are equal and are written as CPCT (Corresponding part of the congruent triangle).

Criteria for Congruency

The following are the criteria for the congruency of the triangles.

SSS Criteria for Congruency

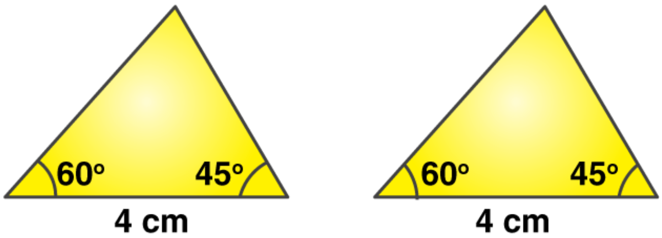
- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- If all sides are exactly the same, then their corresponding angles must also be exactly the same.

SAS Criteria for Congruency

– Axiom: Two triangles are congruent if two sides and the **included** angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.

ASA Criteria for Congruency

– Two triangles are congruent if two angles and the **included** side of one triangle are equal to the corresponding two angles and the included side of the other triangle



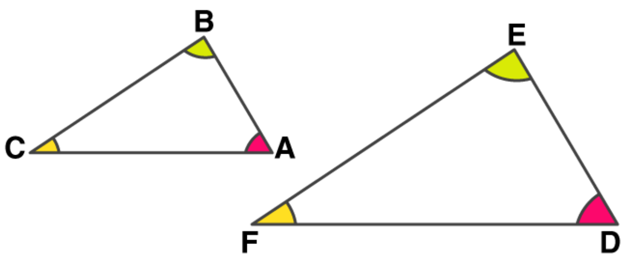
An included side is 4cm

AAS Criteria for Congruency

– Two triangles are said to be congruent to each other if two angles and one side of one triangle are equal to two angles and one side of the other triangle.

Why SSA and AAA congruency rules are not valid?

- SSA or ASS test is not a valid test for congruency as the angle is not included between the pairs of equal sides.-
- The AAA test also is not a valid test as even though 2 triangles can have all three same angles, the sides can be of differing lengths. This becomes a test for similarity (AA).



Angles of a triangle

RHS Criteria for Congruency

- If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.
- RHS stands for Right angle – Hypotenuse – Side.

Properties of Isosceles triangle

– If 2 sides of the triangle are equal, the angles opposite those sides are also equal and vice versa.

Criteria for Congruency of triangles

– The criteria for congruency of triangles are :

- SAS
- ASA
- AAS
- SSS
- RHS

symbolically, it is expressed as ΔABC≅ΔXYZ

Inequalities in Triangles

Relationship between unequal sides of the triangle and the angles opposite to it.

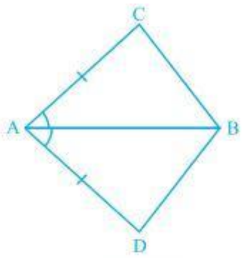
– If 2 sides of a triangle are unequal, then the angle opposite to the longer side will be larger than the angle opposite to the shorter side.

Triangle inequality

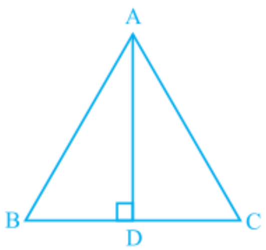
– The sum of the lengths of any two sides of a triangle must be greater than the third side.

Assignment:

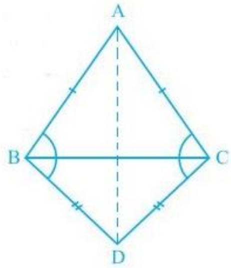
1. In quadrilateral ACBD, AC = AD and AB bisect A (see Fig.). Show that D ABC @ D ABD.. What can you say about BC and BD?



2. AB is a line segment and line *l* is its perpendicular bisector. If a point P lies on *l*, show that P is equidistant from A and B.
3. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig) .Show that ABC @ D CDA..
4. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that Ð BAD = Ð ABE and Ð EPA = Ð DPB (see Fig.). Show that
   (i) D DAP @ D EBP
   (ii) AD = BE
5. In ΔABC, AD is the perpendicular bisector of BC (see Fig). Show that ΔABC is an isosceles triangle in which AB = AC.

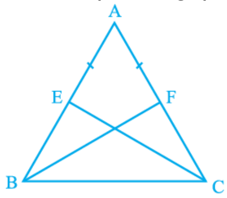


6. ABC and DBC are two isosceles triangles on the same base BC (see Fig.).

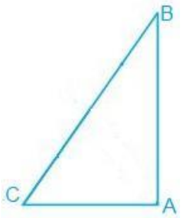


Show that ∠ ABD = ∠ ACD.

7. E and F are respectively the mid-points of equal sides AB and AC of D ABC (see Fig.) Show that BF = CE.

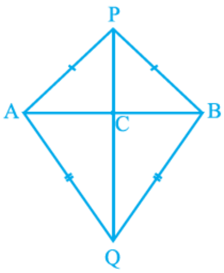


8. ABC is a right-angled triangle in which A = 90° and AB = AC. Find B and C.



9. Show that the angles of an equilateral triangle are 60° each.

10. AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (see Fig.).Show that the line PQ is the perpendicular bisector of AB.

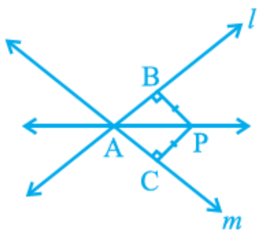


11. D is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC (ii) AD bisects A.

12. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

13. P is a point equidistant from two lines l and m intersecting at point A (see Fig.). Show that the line AP bisects the angle between them.



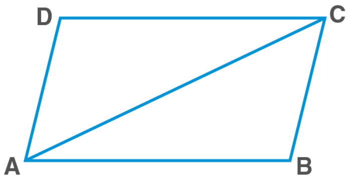
14. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠ B = ∠ C.

Chapter- Quadrilaterals

INTRODUCTION

Parallelogram:

Opposite sides of a parallelogram are equal



In ΔABC and ΔCDA

AC=AC [Common / transversal]

∠BCA=∠DAC [alternate angles]

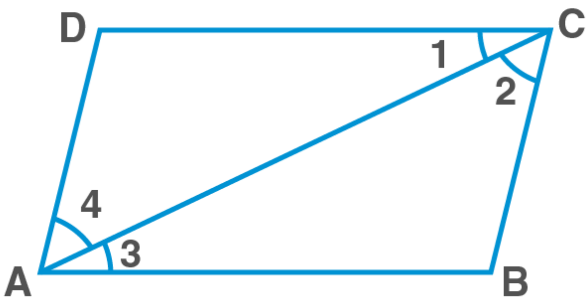
∠BAC=∠DCA [alternate angles]

ΔABC≅ΔCDA [ASA rule]

Hence,

AB=DC and AD=BC [C.P.C.T.C]

Opposite angles in a parallelogram are equal



In parallelogram ABCD

AB∥CD; and AC is the transversal

Hence, ∠1=∠3….(1) (alternate interior angles)

BC∥DA; and AC is the transversal

Hence, ∠2=∠4….(2) (alternate interior angles)

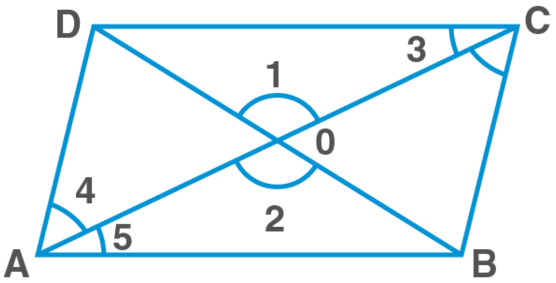
Adding (1) and (2)

$\angle 1+\angle 2=\angle 3+\angle 4$

$\angle BAD=\angle BCD$

Similarly,
$\angle ADC=\angle ABC$

– **Diagonals of a parallelogram bisect each other.**



In ΔAOB and ΔCOD,

$\angle 3=\angle 5$ [alternate interior angles]

$\angle 1=\angle 2$ [vertically opposite angles]

AB=CD [opp. Sides of parallelogram]

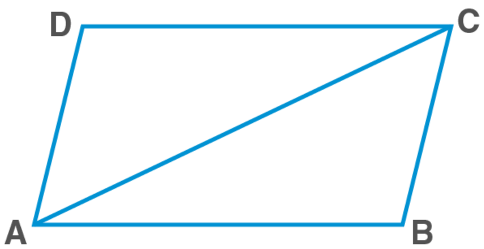
$\Delta AOB\cong\Delta COD$ [AAS rule]

OB=OD and OA=OC [C.P.C.T]

Hence, proved

Conversely,
– **If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.**

– **Diagonal of a parallelogram divides it into two congruent triangles.**



In ΔABC and ΔCDA,

AB=CD [Opposite sides of parallelogram]

BC=AD [Opposite sides of parallelogram]

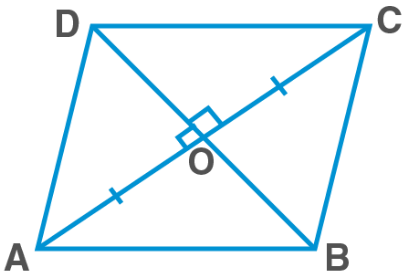
AC=AC [Common side]

ΔABC≅ΔCDA [by SSS rule]

Hence, proved.

Diagonals of a rhombus bisect each other at right angles

Diagonals of a rhombus bisect each – other at right angles



In ΔAOD and ΔCOD,

OA=OC [Diagonals of parallelogram bisect each other]

OD=OD [Common side]

AD=CD [Adjacent sides of a rhombus]

ΔAOD≅ΔCOD [SSS rule]

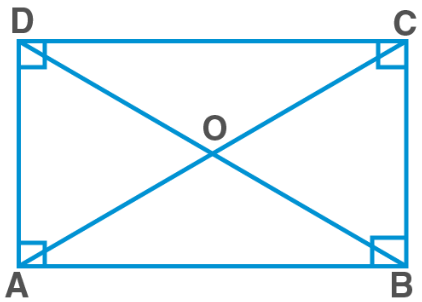
∠AOD=∠DOC [C.P.C.T]

∠AOD+∠DOC=180 [∵ AOC is a straight line]

Hence, ∠AOD=∠DOC=90

Hence proved.

Diagonals of a rectangle bisect each other and are equal



Rectangle ABCD

In ΔABC and ΔBAD,

AB=BA [Common side]

BC=AD [Opposite sides of a rectangle]

∠ABC=∠BAD [Each = 90^0 ∵ ABCD is a Rectangle]

ΔABC≅ΔBAD [SAS rule]

∴AC=BD [C.P.C.T]

Consider ΔOAD and ΔOCB,

AD=CB [Opposite sides of a rectangle]

∠OAD=∠OCB [∵ AD||BC and transversal AC intersects them]

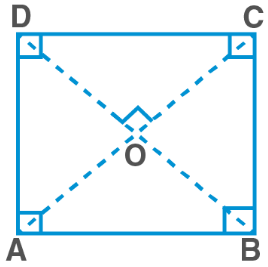
∠ODA=∠OBC [∵ AD||BC and transversal BD intersects them]

ΔOAD≅ΔOCB [ASA rule]

∴OA=OC [C.P.C.T]

Similarly we can prove OB=OD

Diagonals of a square bisect each other at right angles and are equal



Square ABCD

In ΔABC and ΔBAD,

AB=BA [Common side]

BC=AD [Opposite sides of a Square]

∠ABC=∠BAD [Each = 90^0 ∵ ABCD is a Square]

ΔABC≅ΔBAD [SAS rule]

$\therefore AC=BD$ [C.P.C.T]

Consider ΔOAD and ΔOCB,

$AD=CB$ [Opposite sides of a Square]

$\angle OAD=\angle OCB$ [$\because$ AD||BC and transversal AC intersects them]

$\angle ODA=\angle OBC$ [$\because$ AD||BC and transversal BD intersects them]

$\Delta OAD\cong\Delta OCB$ [ASA rule]

$\therefore OA=OC$ [C.P.C.T]

Similarly we can prove $OB=OD$

In ΔOBA and ΔODA,

$OB=OD$ [proved above]

$BA=DA$ [Sides of a Square]

$OA=OA$ [Common side]

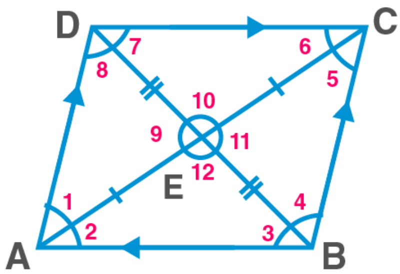
$\Delta OBA\cong\Delta ODA$, [SSS rule]

$\therefore \angle AOB=\angle AOD$ [C.P.C.T]

But, $\angle AOB+\angle AOD=180^0$ [Linear pair]

$\therefore \angle AOB=\angle AOD=90^0$

Important results related to parallelograms



Parallelogram ABCD

Opposite **sides** of a parallelogram are **parallel** and **equal**.

AB||CD,AD||BC,AB=CD,AD=BC

Opposite **angles** of a parallelogram are **equal** adjacent angels are **supplementary**.

$\angle A=\angle C, \angle B=\angle D,$

$\angle A+\angle B=180^0, \angle B+\angle C=180^0, \angle C+\angle D=180^0, \angle D+\angle A=180^0$

A **diagonal** of parallelogram divides it into **two congruent triangles**.

$\Delta ABC \cong \Delta CDA$ [With respect to AC as diagonal]

$\Delta ADB \cong \Delta CBD$ [With respect to BD as diagonal]

The diagonals of a parallelogram **bisect** each other.

$AE=CE, BE=DE$

$\angle 1=\angle 5$ (alternate interior angles)

$\angle 2=\angle 6$ (alternate interior angles)

$\angle 3=\angle 7$ (alternate interior angles)

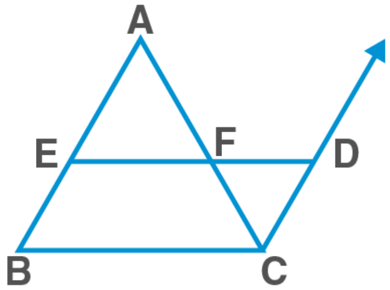
$\angle 4=\angle 8$ (alternate interior angles)

$\angle 9=\angle 11$ (vertically opp. angles)

$\angle 10=\angle 12$ (vertically opp. angles)

The Mid-Point Theorem

The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side



In ΔABC, E – the midpoint of AB; F – the midpoint of AC

Construction: Produce EF to D such that EF=DF.

In ΔAEF and ΔCDF,

AF=CF [F is the midpoint of AC]

∠AFE=∠CFD [V.O.A]

EF=DF [Construction]

∴ΔAEF≅ΔCDF [SAS rule]

Hence,

∠EAF=∠DCF….(1)

DC=EA=EB [E is the midpoint of AB]

DC∥EA∥AB [Since, (1), alternate interior angles]

DC∥EB

So EBCD is a parallelogram

Therefore, BC=ED and BC∥ED

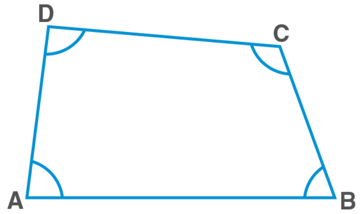
Since, ED=EF+FD=2EF=BC [∵ EF=FD]

We have,EF=12BC and EF||BC

Hence proved.

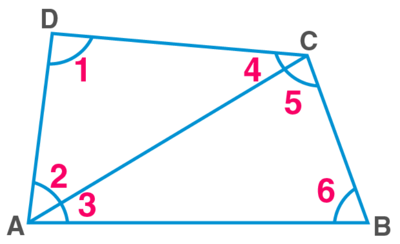
Quadrilaterals

Any four points in a plane, of which three are non-collinear are joined in order results into a four-sided closed figure called **‘quadrilateral’**



Quadrilateral

Angle sum property of a quadrilateral



Angle sum property – Sum of angles in a quadrilateral is 360

In △ADC,

∠1+∠2+∠4=180 (Angle sum property of triangle)……………(1)

In △ABC,

∠3+∠5+∠6=180 (Angle sum property of triangle)………………(2)

(1) + (2):

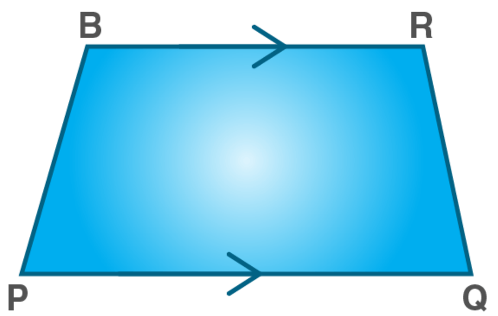
∠1+∠2+∠3+∠4+∠5+∠6=360

I.e, ∠A+∠B+∠C+∠D=360

Hence proved

Types of Quadrilaterals
Trapezium

A **trapezium** is a quadrilateral with any **one pair of opposite sides parallel**.

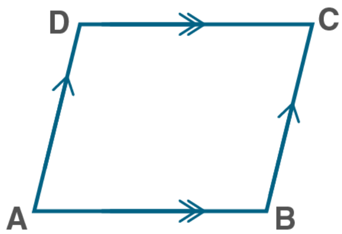


Trapezium

PQRS is a trapezium in which PQ||RS

Parallelogram

A **parallelogram** is a quadrilateral, with both pair of **opposite sides parallel and equal**. In a parallelogram, diagonals bisect each other.

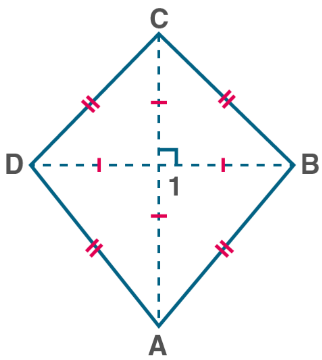


Parallelogram ABCD

Parallelogram ABCD in which AB||CD,BC||AD and AB=CD,BC=AD

Rhombus

A **rhombus** is a parallelogram with **all sides equal.** In a rhombus, diagonals bisect each other perpendicularly

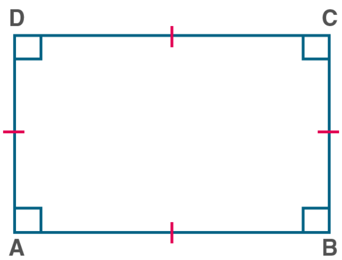


Rhombus ABCD

A rhombus ABCD in which AB=BC=CD=AD and AC⊥BD

Rectangle

A **rectangle** is a parallelogram with **all angles as right angles**.

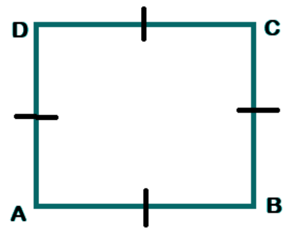


Rectangle ABCD

A rectangle ABCD in which, ∠A=∠B=∠C=∠D=900

Square

A **square** is a special case of a parallelogram with **all angles as right angles and all sides equal.**

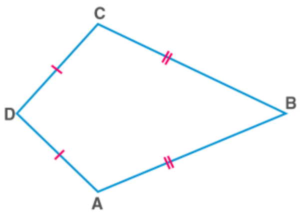


Square ABCD

A square ABCD in which ∠A=∠B=∠C=∠D=900 and AB=BC=CD=AD

Kite

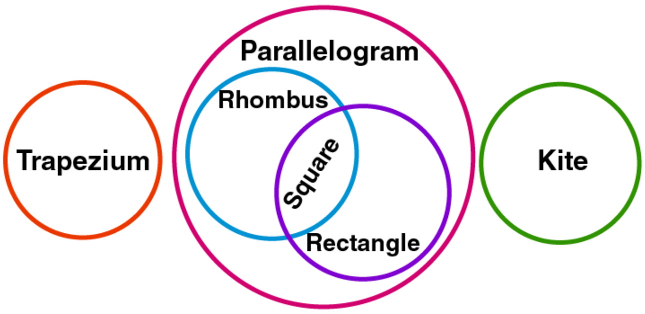
A **kite** is a quadrilateral with **adjacent sides equal**.



Kite ABCD

A kite ABCD in which AB=BC and AD=CD

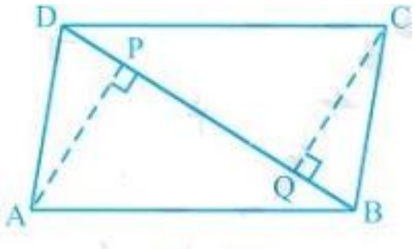
Venn diagram for different types of quadrilaterals



Assignment:

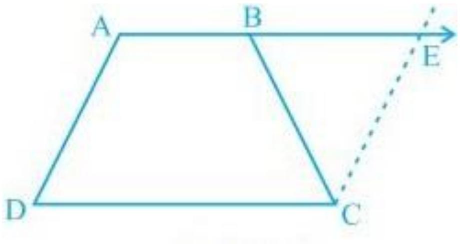
1. Show that each angle of a rectangle is a right angle.

2. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.
3. Show that the diagonals of a square are equal and bisect each other at right angles.
4. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that (i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$

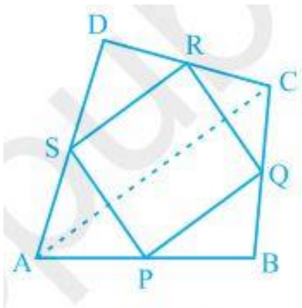


5. ABCD is a trapezium in which AB || CD and AD = BC (see Fig.). Show that (i) $\angle A = \angle B$ (ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$ (iv) diagonal AC = diagonal BD

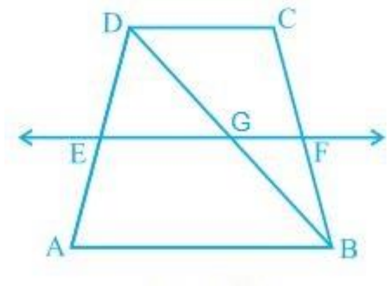


6. Show that the bisectors of angles of a parallelogram form a rectangle.
7. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that:
(i) SR || AC and SR = 1/2 AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.



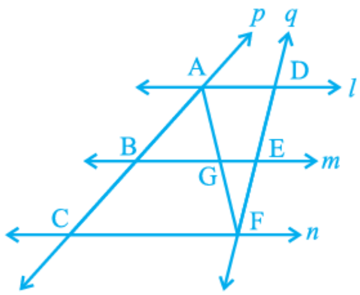
8. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

9. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.



10. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) MD ⊥ AC
(iii) CM = MA = ½ AB

11. *l*, *m* and *n* are three parallel lines intersected by transversals *p* and *q* such that *l*, *m* and *n* cut off equal intercepts AB and BC on *p* (see Fig.)Show that *l*, *m* and *n* cut off equal intercepts DE and EF on *q* also.



12. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Chapter- Circles

INTRODUCTION

Circles

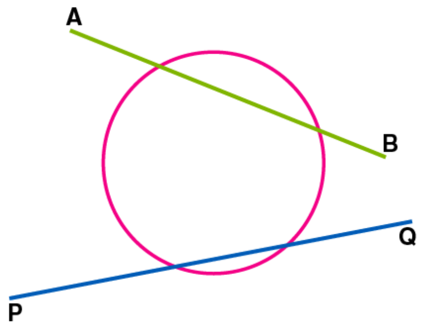
- The set of all the points in a plane that is at a fixed distance from a fixed point makes a circle.
- A Fixed point from which the set of points are at fixed distance is called the centre of the circle.
- A circle divides the plane into 3 parts: interior (inside the circle), the circle itself and exterior (outside the circle)

Radius

– The distance between the centre of the circle and any point on its edge is called the radius.

Tangent and Secant

A line that touches the circle at exactly one point is called it’s tangent. A line that cuts a circle at two points is called a secant.



In the above figure: PQ is the tangent and AB is the secant.

Chord

-The line segment within the circle joining any 2 points on the circle is called the chord.

Diameter

– A Chord passing through the centre of the circle is called the diameter. – The Diameter is 2 times the radius and it is the longest chord.

Arc

– The portion of a circle(curve) between 2 points is called an arc. – Among the two pieces made by an arc, the longer one is called a major arc and the shorter one is called a minor arc.

Circumference

The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula $2\pi r$

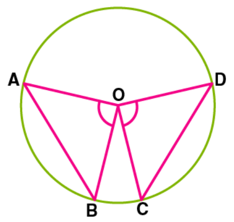
Segment and Sector

– A circular segment is a region of a circle which is “cut off” from the rest of the circle by a secant or a chord. – Smaller region cut off by a chord is called minor segment and the bigger region is called major segment. –

-A sector is the portion of a circle enclosed by two radii and an arc, where the smaller area is known as the minor sector and the larger being the major sector.– For 2 equal arcs or for semicircles – both the segment and sector is called the semicircular region.

Theorem of equal chords subtending angles at the centre.

– Equal chords subtend equal angles at the centre.



Proof: AB and CD are the 2 equal chords.

In Δ AOB and Δ COD

OB = OC [Radii]

OA = OD [Radii]

AB = CD [Given]

ΔAOB ≅ ΔCOD (SSS rule)

Hence, ∠AOB = ∠COD [CPCT]

Theorem of equal angles subtended by different chords.

– If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.

Proof: In ΔAOB and ΔCOD

OB = OC [Radii] ∠AOB=∠COD [Given]

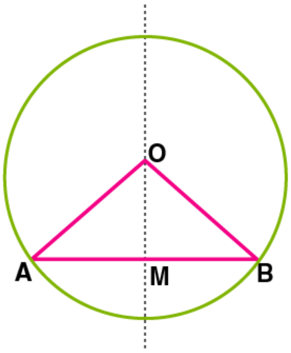
OA = OD [Radii]

ΔAOB ≅ ΔCOD (SAS rule)

Hence, AB=CD [CPCT]

Perpendicular from the centre to a chord bisects the chord.

Perpendicular from the centre of a circle to a chord bisects the chord.



Proof: AB is a chord and OM is the perpendicular drawn from the centre.

From ΔOMB and ΔOMA,

$\angle OMA = \angle OMB = 90^0$ OA = OB (radii)

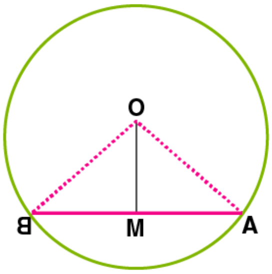
OM = OM (common)

Hence, ΔOMB ≅ ΔOMA (RHS rule)

Therefore AM = MB [CPCT]

A Line through the centre that bisects the chord is perpendicular to the chord.

– A line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Proof: OM drawn from the center to bisect chord AB.

From ΔOMA and ΔOMB,

OA = OB (Radii)

OM = OM (common)

AM = BM (Given)

Therefore, ΔOMA ≅ ΔOMB (SSS rule)

$\Rightarrow \angle OMA = \angle OMB$ (C.P.C.T)

But, $\angle OMA + \angle OMB = 180^0$

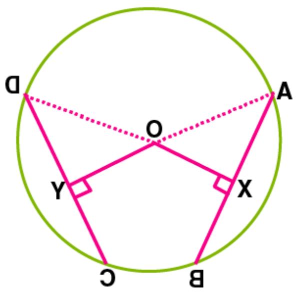
Hence, $\angle OMA = \angle OMB = 90^0 \Rightarrow OM \perp AB$

Circle through 3 points

– There is one and only one circle passing through three given noncollinear points. – A unique circle passes through 3 vertices of a triangle ABC called as the circumcircle. The centre and radius are called the circumcenter and circumradius of this triangle, respectively.

Chords equidistant from the centre are equal

Chords equidistant from the centre of a circle are equal in length.



Proof: Given OX = OY (The chords AB and CD are at equidistant) OX⊥AB, OY⊥CD

In ΔAOX and ΔDOY

∠OXA =∠OYD (Both 90^0)

OA = OD (Radii)

OX = OY (Given)

ΔAOX ≅ ΔDOY (RHS rule)

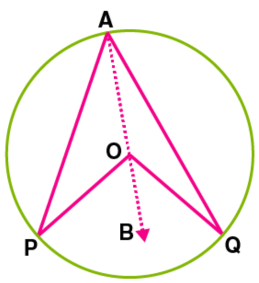
Therefore AX = DY (CPCT)

Similarly XB = YC

So, AB = CD

The angle subtended by an arc of a circle on the circle and at the centre

The angle subtended by an arc at the centre is double the angle subtended by it on any part of the circle.



Angles in the same segment of a circle.

–Angles in the same segment of a circle are equal.

The angle subtended by diameter on the circle

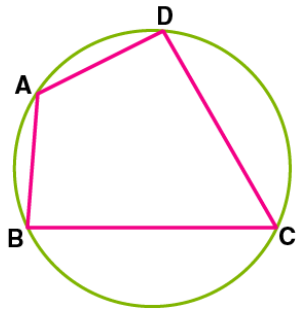
– Angle subtended by diameter on a circle is a right angle. (Angle in a semicircle is a right angle)

Line segment that subtends equal angles at two other points

– If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e they are concyclic).

Cyclic Quadrilateral

– Quadrilateral is called a cyclic quadrilateral if all the four vertices lie on a circle.



In a circle, if all four points A, B, C and D lie on the circle, then quadrilateral ABCD is a cyclic quadrilateral.

Sum of opposite angles of a cyclic quadrilateral

– If the sum of a pair of opposite angles of a quadrilateral is 180 degree, the quadrilateral is cyclic.

Sum of pair of opposite angles in a quadrilateral

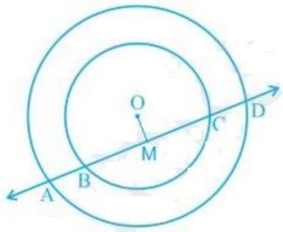
– The sum of either pair of opposite angles of a cyclic quadrilateral is 180 degree.

Assignment:

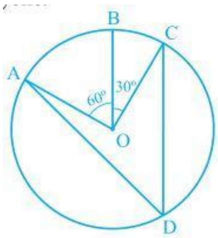
1. Fill in the blanks:

   (i) The centre of a circle lies in _____________ of the circle. (exterior/ interior)
   (ii) A point, whose distance from the centre of a circle is greater than its radius lies in ___________ of the circle. (exterior/ interior)
   (iii) The longest chord of a circle is a ______________ of the circle.
   (iv) An arc is a ____________ when its ends are the ends of a diameter.
   (v) Segment of a circle is the region between an arc and ______________ of the circle.
   (vi) A circle divides the plane, on which it lies, in ______________ parts.
2. Write True or False: Give reasons for your Solution.
   (i)Line segment joining the centre to any point on the circle is a radius of the circle.
   (ii)A circle has only finite number of equal chords.
   (iii) If a circle is divided into three equal arcs, each is a major arc.
   (iv)A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
3. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.
4. *If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.*
5. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.
6. Suppose you are given a circle. Give a construction to find its centre.

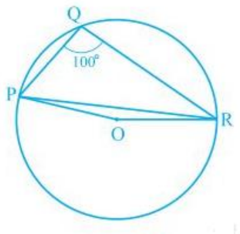
7. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.
8. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.
9. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.
10. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.
11. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.
12. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig)



13. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.
14. In Fig, A,B and C are three points on a circle with centre O such that BOC = 30° and AOB = 60°. If D is a point on the circle other than the arc ABC, find ADC.



15. In Fig., PQR = 100°, where P, Q and R are points on a circle with centre O. Find OPR.



16. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.
17. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.
18. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Chapter- Constructions

INTRODUCTION

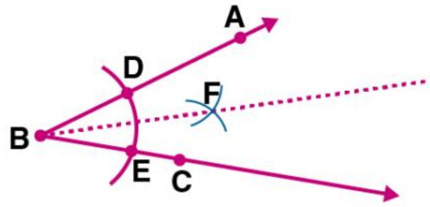
Linear Pair axiom

- If a ray stands on a line then the adjacent angles form a linear pair of angles.
- If two angles form a linear pair, then uncommon arms of both the angles form a straight line.

Construction of an Angle bisector

Suppose we want to draw the angle bisector of ∠ABC we will do it as follows:

- Taking B as centre and any radius, draw an arc to intersect AB and BC to intersect at D and E respectively.
- Taking D and E as centres and with radius more than DE/2, draw arcs to intersect each other at a point F.
- Draw the ray BF. This ray BF is the required bisector of the ∠ABC.

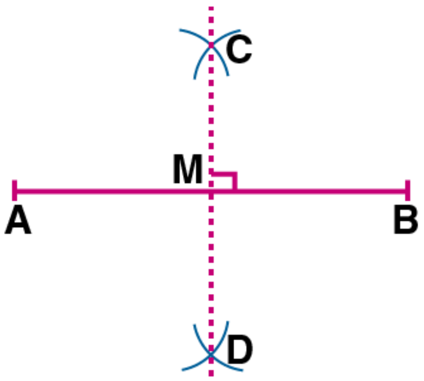


Perpendicular Bisector

Construction of a perpendicular bisector

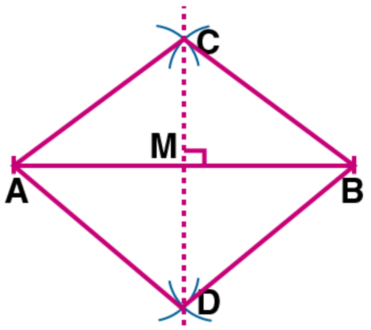
Steps of construction of a perpendicular bisector on the line segment AB:

- Take A and B as centres and radius more than AB/2 draw arcs on both sides of the line.
- Arcs intersect at the points C and D. Join CD.
- CD intersects AB at M. CMD is the required perpendicular bisector of the line segment AB.



Proof of validity of construction of a perpendicular bisector

Proof of the validity of construction of the perpendicular bisector:
ΔDAC and ΔDBC are congruent by SSS congruency. (∵ AC = BC, AD = BD and CD = CD)
∠ACM and ∠BCMare equal (cpct)
ΔAMC and ΔBMC are congruent by SAS congruency. (∵ AC = BC, ∠ACM = ∠BCM and CM = CM)
AM = BM and ∠AMC = ∠BMC (CPCT)
∠AMC + ∠BMC = 180^0 (Linear Pair Axiom)
∴ ∠AMC = ∠BMC = 90°
Therefore, CMD is the perpendicular bisector.

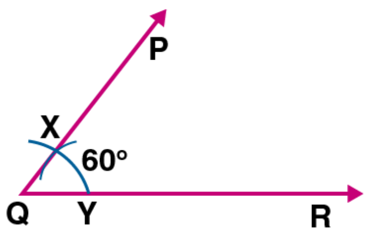


Constructing Angles

Construction of an Angle of 60 degrees

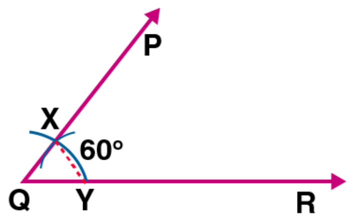
Steps of construction of an angle of 60 degrees:

- Draw a ray QR.
- Take Q as the centre and some radius draw an arc of a circle, which intersects QR at a point Y.
- Take Y as the centre with the same radius draw an arc intersecting the previously drawn arc at point X.
- Draw a ray QP passing through X
- ∠PQR=60°



Proof for the validity of construction of an Angle of 60 degrees

Proof for the validity of construction of the 60^0 angle:
Join XY
XY = XQ = YQ (By construction)
∴△XQY is an equilateral triangle.
Therefore, ∠XQY = ∠PQR = 60^0



Triangle Constructions

Construction of triangles

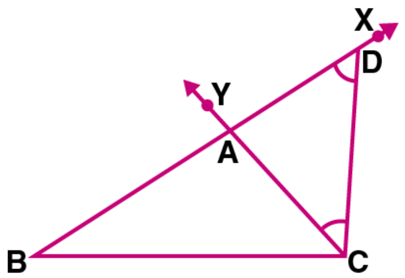
At least three parts of a triangle have to be given for constructing it but not all combinations of three parts are sufficient for the purpose.
Therefore a unique triangle can be constructed if the following parts of a triangle are given:

- two sides and the included angle is given.
- three sides are given.
- two angles and the included side is given.
- In a right triangle, hypotenuse and one side are given.
- If two sides and an angle (not the included angle) are given, then it is not always possible to construct such a triangle uniquely.

Given base, base angle and sum of other two sides

Steps for construction of a triangle given base, base angle, and the sum of other two sides:

- Draw the base BC and at point B make an angle say XBC equal to the given angle.
- Cut the line segment BD equal to AB + AC from ray BX.
- Join DC and make an angle DCY equal to ∠BDC.
- Let CY intersect BX at A.
- ABC is the required triangle.

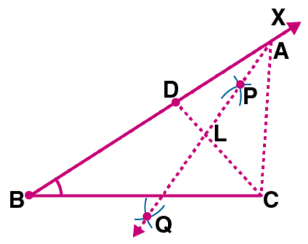


Given base(BC), base angle(ABC) and AB-AC

Steps of construction of a triangle given base(BC), base angle(∠ABC) and difference of the other two sides (AB-AC):

- Draw base BC and with point B as the vertex make an angle XBC equal to the given angle.
- Cut the line segment BD equal to AB – AC(AB > AC) on the ray BX.
- Join DC and draw the perpendicular bisector PQ of DC.
- Let it intersect BX at a point A. Join AC.

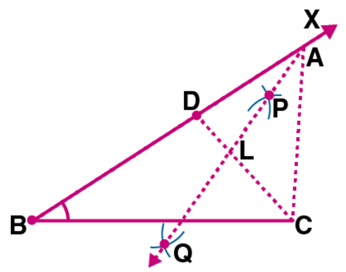
- Then △ABC is the required triangle.



Proof for validation for Construction of a triangle with given base, base angle and difference between two sides

Validation of the steps of construction of a triangle with given base, base angle and difference between two sides

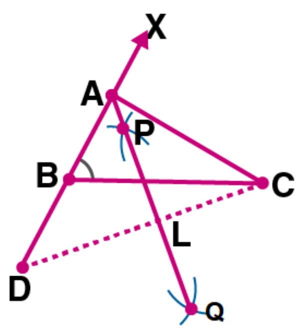
- Base BC and ∠B are drawn as given.
- Point A lies on the perpendicular bisector of DC. So, AD = AC.
- BD = AB – AD = AB – AC (∵ AD = AC).
- Therefore ABC is the required triangle.



Given base (BC), base angle (ABC) and AC-AB

Steps of construction of a triangle given base (BC), base angle (∠ABC) and difference of the other two sides (AC-AB):

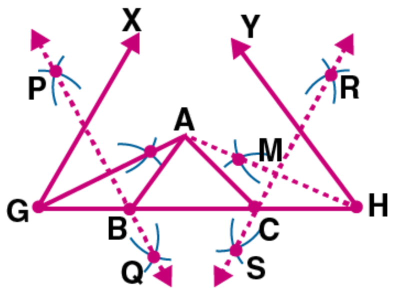
- Draw the base BC and at point B make an angle XBC equal to the given angle.
- Cut the line segment BD equal to AC – AB from the line BX extended on the opposite side of line segment BC.
- Join DC and draw the perpendicular bisector, say PQ of DC.
- Let PQ intersect BX at A. Join AC.
- △ABC is the required triangle.



Given perimeter and two base angles

Steps of construction of a triangle with given perimeter and two base angles.

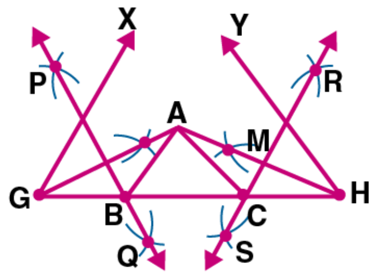
- Draw a line segment, say GH equal to BC + CA + AB.
- Make angles XGH equal to ∠B and YHG equal to ∠C, where angle B and C are the given base angles.
- Draw the angle bisector of∠XGH and ∠YHG. Let these bisectors intersect at a point A.
- Draw perpendicular bisectors PQ of AG and RS of AH.
- Let PQ intersect GH at B and RS intersect GH at C. Join AB and AC
- △ABC is the required triangle.



Proof for validation for Construction of a triangle with given perimeter and two base angles

Validating the steps of construction of a triangle with given perimeter and two base angles:

- B lies on the perpendicular bisector PQ of AG and C lies on the perpendicular bisector RS of AH. So, GB = AB and CH = AC.
- BC + CA + AB = BC + GB + CH = GH (∵ GB = AB and CH = AC)
- ∠BAG = ∠AGB (∵ΔAGB, AB = GB)
- ∠ABC = ∠BAG + ∠AGB = 2∠AGB = ∠XGH
- Similarly, ∠ACB =∠YHG



Assignment:

1. Construct an angle of 90° at the initial point of a given ray and justify the construction.
2. Construct an angle of 45° at the initial point of a given ray and justify the construction.

3. **Construct the angles of the following measurements:**

i) 30° (ii) $22\frac{1}{2}^{\circ}$ (iii) 15°

4. Construct the following angles and verify by measuring them by a protractor:

   (i) 75° (ii) 105° (iii) 135°

5. Construct an equilateral triangle, given its side and justify the construction.
6. Construct a triangle ABC in which BC = 7cm, ∠B = 75° and AB+AC = 13 cm.
7. Construct a triangle ABC in which BC = 8cm, ∠B = 45° and AB–AC = 3.5 cm.
8. Construct a triangle PQR in which QR = 6cm, ∠Q = 60° and PR–PQ = 2cm.

Chapter- Heron's Formula

INTRODUCTION

Triangle

The plane closed figure, with three sides and three angles is called as a triangle.

Types of triangles:
Based on **sides** – a) Equilateral b) Isosceles c) Scalene
Based on **angles** – a) Acute angled triangle b) Right-angled triangle c) Obtuse angled triangle

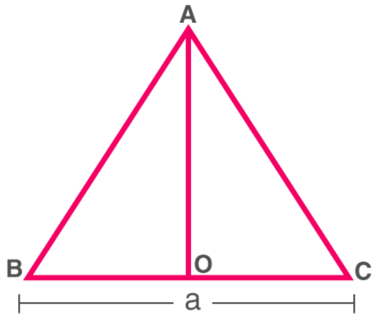
Area of a triangle

Area = (1/2) × base × height

In case of equilateral and isosceles triangles, if the lengths of the sides of triangles are given then, we use Pythagoras theorem in order to find the height of a triangle.

Area of an equilateral triangle

Consider an equilateral ΔABC, with each side as a unit. Let AO be the perpendicular bisector of BC. In order to derive the formula for the area of an equilateral triangle, we need to find height AO.



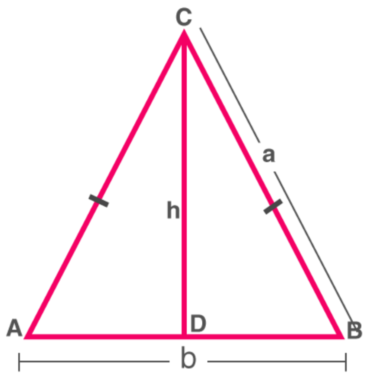
Using Pythagoras theorem,
AC2 = OA2 + OC2
OA2 = AC2 − OC2
Substitute AC = a,OC = a/2 in the above equation.
OA2 = a2 − a2/4
OA = √3a/2

We know that the area of the triangle is:
A = (1/2) × base × height
A = (1/2) × a × (√3a/2)

∴ Area of Equilateral triangle = √3a2/4

Area of an isosceles triangle

Consider an isosceles ΔABC with equal sides as a units and base as b units.



Isosceles triangle ABC

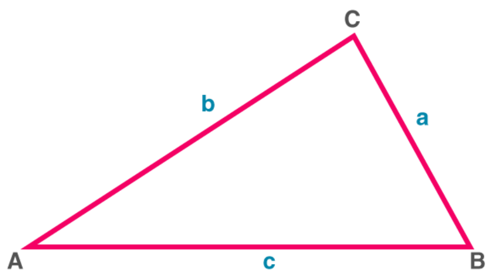
The height of the triangle can be found by Pythagoras' Theorem :
$CD^2 = AC^2 - AD^2$
$\Rightarrow h^2 = a^2 - (b^2/4) = (4a^2 - b^2)/4$

$\Rightarrow h = (1/2)\sqrt{4a^2 - b^2}$
Area of triangle is $A = (1/2)bh$
$\therefore A = (1/2) \times b \times (1/2)\sqrt{4a^2 - b^2}$

$\therefore A = (1/4) \times b \times \sqrt{4a^2 - b^2}$

Area of a triangle – By Heron's formula

Area of a ΔABC, given sides a, b, c by **Heron's formula** (also known as Hero's Formula) is:



Triangle ABC

Find semi perimeter $(s) = (a + b + c)/2$

Area $= \sqrt{s(s - a)(s - b)(s - c)}$

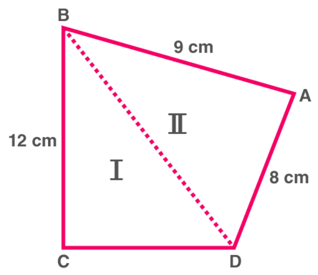
This formula is helpful to find the area of a scalene triangle, given the lengths of all its sides.

Area of any polygon – By Heron's formula

For a quadrilateral, when one of its diagonal value and the sides are given, the area can be calculated by **splitting the given quadrilateral into two triangles and use the Heron's formula.**

Example :**A park, in the shape of a quadrilateral ABCD, has ∠C=90∘, AB = 9 cm, BC = 12 cm, CD = 5 cm and AD = 8 cm. How much area does it occupy?**

⇒We draw the figure according to the information given.



The figure can be split into 2 triangles ΔBCD and ΔABD
From ΔBCD, we can find BD (Using Pythagoras' Theorem)
BD2 = 122 + 52 = 169
BD = 13cm
Semi-perimeter for ΔBCD S1 = (12 + 5 + 13)/2 = 15

Semi-perimeter ΔABD S2 = (9 + 8 + 13)/2 = 15

Using Heron's formula A1 and A2 will be:

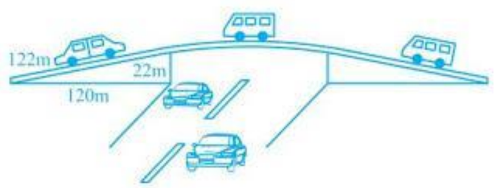
A1 = √[15(15 – 12)(15 – 5)(15 – 13)]

A1 = √(15 × 3 × 10 × 2)
A1 =√900 = 30 cm2
Similarly,

A2 will be 35.49 cm2.
The area of the quadrilateral ABCD = A1 + A2 = 65.49 cm^2

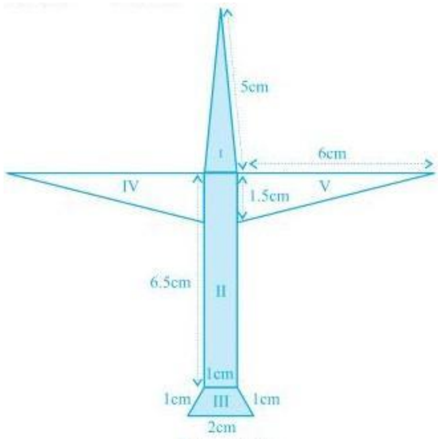
Assignment:

1. Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm.
2. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?
3. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig.). The advertisements yield an earning of ₹5000 per m^2 per year. A company hired one of its walls for 3 months. How much rent did it pay?



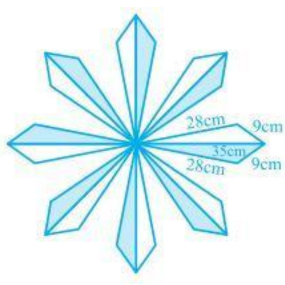
4. Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.
5. An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.
6. A triangular park ABC has sides 120m, 80m and 50m (see Fig. 12.7). A gardener *Dhania* has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of `20 per metre leaving a space 3m wide for a gate on one side.

7. A park, in the shape of a quadrilateral ABCD, has C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?
8. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.
9. Radha made a picture of an aeroplane with coloured paper as shown in Fig Find the total area of the paper used.



10. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

11. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.
12. Kamla has a triangular field with sides 240 m, 200 m, 360 m, where she grew wheat. In another triangular field with sides 240 m, 320 m, 400 m adjacent to the previous field, she wanted to grow potatoes and onions (see Fig. 12.11). She divided the field in two parts by joining the mid-point of the longest side to the opposite vertex and grew patatoes in one part and onions in the other part. How much area (in hectares) has been used for wheat, potatoes and onions? (1 hectare = 10000 m2)
13. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig.). Find the cost of polishing the tiles at the rate of 50p per cm^2 .



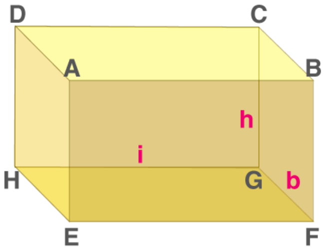
Chapter- Surface Areas and Volumes

INTRODUCTION

Cuboid

A cuboid is a three dimensional Shape. The cuboid is made from six rectangular faces, which are placed at right angles. The total surface area of a cuboid is equal to the sum of the areas of its six rectangular faces.

Total Surface Area of a Cuboid



Consider a cuboid whose length is "*l*" cm, breadth is *b* cm and height *h* cm.

Area of face ABCD = Area of Face EFGH = (l × b) cm^2

Area of face AEHD = Area of face BFGC = (b × h) cm^2

Area of face ABFE = Area of face DHGC = (l × h) cm^2

Total surface area (TSA) of cuboid = Sum of the areas of all its six faces
= 2(l × b) + 2(b × h) + 2(l × h)
TSA (cuboid)= 2(lb + bh +lh)

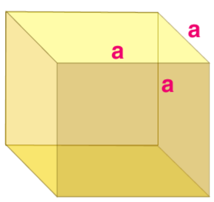
Lateral Surface Area of a Cuboid

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
The lateral surface area of the cuboid
= Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC
= 2(b × h) + 2(l × h)
LSA (cuboid) = 2h(l + b)

Cube

A **cuboid** whose length, breadth and height are all **equal**, is called a **cube**. It is a three-dimensional shape bounded by **six equal squares**. It has 12 edges and 8 vertices.

Total Surface Area of a cube



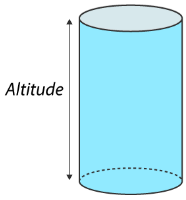
For cube, length = breadth = height
Suppose the length of an edge = a
Total surface area(TSA) of the cube = 2(a × a + a × a + a × a)
TSA(cube) = 2 × ($3a^2$) = $6a^2$

Lateral Surface area of a cube

Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
Lateral surface area of cube = $2(a \times a + a \times a) = 4a^2$

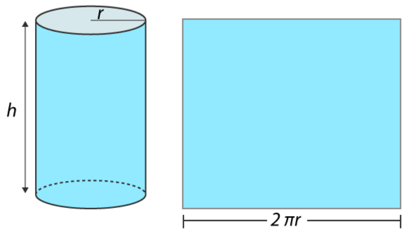
Right Circular Cylinder

A right circular cylinder is a closed solid that has two parallel circular bases connected by a curved surface in which the two bases are exactly over each other and the axis is at right angles to the base.



The curved Surface area of a right circular cylinder

Take a cylinder of base radius *r* and height *h* units. The curved surface of this cylinder, if opened along the diameter (d = 2r) of the circular base will be transformed into a rectangle of length 2πr and height *h* units. Thus,



Curved surface area(CSA) of a cylinder of base radius *r* and height *h* = 2π × r × h

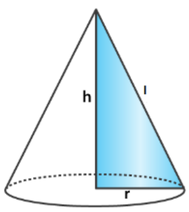
Total surface area of a right circular cylinder

Total surface area(TSA) of a cylinder of base radius *r* and height *h* = 2π × r × h + area of two circular bases
⇒ TSA = 2π × r × h + 2 × πr2
⇒ TSA = 2πr(h + r)

Right Circular Cone

A right circular cone is a circular cone whose axis is perpendicular to its base.

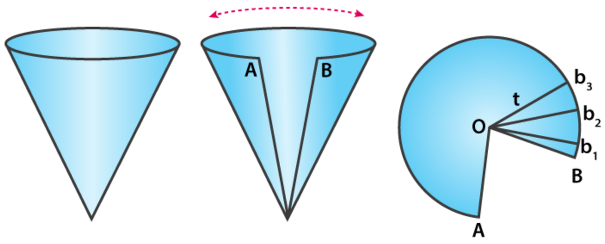
Relation between slant height and height of a right circular cone



The relationship between slant height(l) and height(h) of a right circular cone is:
$l^2 = h^2 + r^2$ (Using Pythagoras Theorem)
Where r is the radius of the base of the cone.

Curved Surface Area of a Right Circular Cone

Consider a right circular cone with slant length l and radius r.
If a perpendicular cut is made from a point on the circumference of the base to the vertex and the cone is opened up, a sector of a circle with radius l is produced as shown in the figure below:



Label A and B and corresponding b_1, b_2 …b_n at equal intervals, with O as the common vertex. The Curved surface area(CSA) of the cone will be the sum of areas of the small triangles: 1/2 × (b_1 + b_2……..b_n) × l
($b_1 + b_2 ... b_n$) is also equal to the circumference of base = $2\pi r$
CSA of right circular cone = $(1/2) \times (2\pi r) \times l = \pi rl$ (On substituting the values)

Total Surface Area of a Right Circular Cone

Total surface area(TSA) = Curved surface area(CSA) + area of base = $\pi rl + \pi r^2 = \pi r(l + r)$

Sphere

A sphere is a closed three-dimensional solid figure, where all the points on the surface of the sphere are equidistant from the common fixed point called “centre”. The equidistant is called the “radius”.

Surface area of a Sphere

The surface area of a sphere of radius r = 4 times the area of a circle of radius r $=4\times(\pi r^2)$
For a sphere Curved surface area (CSA) = Total Surface area(TSA) = $4\pi r^2$

Volume of a Cuboid

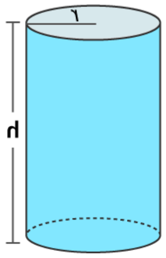
The volume of a cuboid is the product of its dimensions.
Volume of a cuboid = length×breadth×height= lbh
Where l is the length of the cuboid, b is the breadth, and h is the height of the cuboid.

Volume of a Cube

The volume of a cube = base area×height.
Since all dimensions are identical, the volume of the cube = a^3
Where a is the length of the edge of the cube.

Volume of a Right Circular Cylinder

The volume of a right circular cylinder is equal to base area × its height.



The volume of cylinder $=\pi r^2 h$
Where *r* is the radius of the base of the cylinder and *h* is the height of the cylinder.

Volume of a Right Circular Cone

The volume of a Right circular cone is 1/3 times the volume of a cylinder with the same radius and height. In other words, three cones make one cylinder of the same height and base.
The volume of right circular cone = $(1/3)\pi r^2 h$
Where *r* is the radius of the base of the cone and *h* is the height of the cone.

Volume of a Sphere

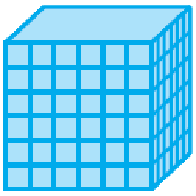
The volume of a sphere of radius *r* $=(4/3)\pi r^3$

Volume and Capacity

The **volume** of an object is the measure of the space it occupies and the **capacity** of an object is the volume of substance its interior can accommodate. The unit of measurement of either volume or capacity is a cubic unit.

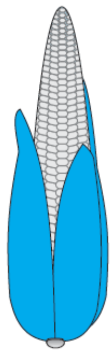
Assignment:

1. Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm (see Fig.). Find how much he would spend for the tiles, if the cost of the tiles is ` 360 per dozen.

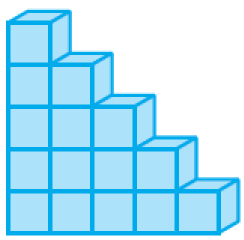


2. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per m^2 is Rs.15000, find the height of the hall.
3. Praveen wanted to make a temporary shelter for her car, by making a box – like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5m, with base dimensions 4m×3m?
4. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 12.50 per m^2.
5. Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope.(see Fig). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius?

6. Find
   (i) the lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5m high.
   (ii) How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank. (Assume π = 22/7)
7. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area (Assume π=22/7)
8. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm. [Use π=3.14]
9. A corn cob (see Fig.), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.



10. The radius of a spherical balloon increases from 7cm to 14cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.
11. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.
12. The hollow sphere, in which the circus motorcyclist performs his stunts,has a diameter of 7 m. Find the area available to the motorcyclist for riding.
13. A matchbox measures 4 cm×2.5cm×1.5cm. What will be the volume of a packet containing 12 such boxes?
14. A child playing with building blocks, which are of the shape of cubes, has built a structure as shown in Fig.. If the edge of each cube is 3 cm, find the volume of the structure built by the child.



15. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m×15 m×6 m. For how many days will the water of this tank last?
16. The capacity of a closed cylindrical vessel of height 1m is15.4 liters. How many square meters of metal sheet would be needed to make it? (Assume π = 22/7)

17. At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see Fig.) of radius 3 cm up to a height of 8 cm, and sold for ` 15 each. How much money does the stall keeper receive by selling the juice completely?
18. The height of a cone is 15cm. If its volume is 1570cm^3, find the diameter of its base. (Use π = 3.14)
19. Monica has a piece of canvas whose area is 551 m2. She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m2, find the volume of the tent that can be made with it.
20. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?
21. A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm3, find the mass of the shot-putt.
22. A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Chapter- Statistics

INTRODUCTION

A study dealing with the collection, presentation and interpretation and analysis of data is called as statistics.

Data

- Facts /figures numerical or otherwise collected for a definite purpose is called as data.
- data collected first-hand data:- Primary
- Secondary data: Data collected from a source that already had data stored

Frequency

– The number of times a particular instance occurs is called frequency in statistics.

Ungrouped data

Ungrouped data is data in its original or raw form. The observations are not classified in groups.

Grouped data

In grouped data, observations are organized in groups.

Class Interval

- The size of the class into which a particular data is divided.
- E.g divisions on a histogram or bar graph.
- **Class width** = upper class limit – lower class limit

Regular and Irregular class interval

- Regular class interval: When the class intervals are equal or of the same sizes.
- E.g 0-10, 10-20, 20-30….. 90-100
- Irregular class interval: When the class intervals are of varying sizes.
- E.g 0-35, 35-45, 45-55, 55- 80, 80-90, 90-95, 95-100

Frequency table

– A frequency table or distribution shows the occurrence of a particular variable in a tabular form.

Sorting

- Raw data needs to be sorted in order to carry out operations.-
- Sorting ⇒ ascending order or descending order

Ungrouped frequency table

– When the frequency of each class interval is not arranged or organised in any manner.

Grouped frequency table

– The frequencies of the corresponding class intervals are organised or arranged in a particular manner, either ascending or descending.

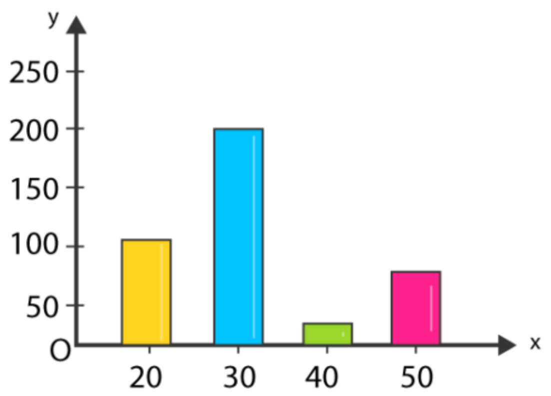
Graphical Representation of Data

Bar graphs

Graphical representation of data using bars of equal width and equal spacing between them (on one axis). The height

Savings (in percentage)	Number of Employees (Frequency)
20	105
30	199
40	29
50	73
Total	400

The data can be represented as:



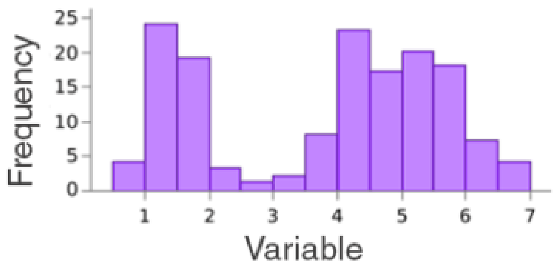
Variable being a number

- A variable can be a number such as ‘no. of students’ or ‘no. of months’.
- Can be represented by bar graphs or histograms depending on the type of data.

Discrete → bar graphs
Continuous → Histograms

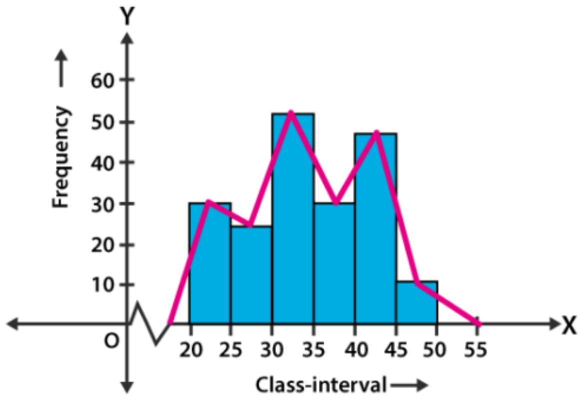
Histograms

- Like bar graphs, but for continuous class intervals.
- Area of each rectangle is ∝ Frequency of a variable and the width is equal to the class interval.



Frequency polygon

- If the midpoints of each rectangle in a histogram are joined by line segments, the figure formed will be a frequency polygon.
- Can be drawn without histogram. Need midpoints of class intervals



Midpoint of class interval

The midpoint of the class interval is called a class mark
Class mark = (Upper limit + Lower limit)/2

Equality of areas

– Addition of two class intervals with zero frequency preceding the lowest class and succeeding the highest class intervals enables to equate the area of the frequency polygon to that of the histogram(Using congruent triangles.)

Measures of Central Tendency

Average

– The average of a number of observations is the sum of the values of all the observations divided by the total number of observations.

Mean

Mean for ungrouped frequency distribution, $\bar{x} = \frac{\sum x_i f_i}{f_i}$

where f_i is the frequency of i^{th} observation x_i

Mode

- The most frequently occurring observation is called the mode.
- The class interval with the highest frequency is the modal class

Median

- Value of the middlemost observation.
- If n(number of observations) is odd, Median =$[(n+1)/2]^{th}$ observation.
- If n is even, the Median is the mean or average of $(n/2)^{th}$ and $[(n+1)/2]^{th}$ observation.

Assignment:

1. The blood groups of 30 students of Class VIII are recorded as follows:
   A, B, O, O, AB, O, A, O, B, A, O, B, A, O, O,
   A, AB, O, A, A, O, O, AB, B, A, O, B, A, B, O.
   Represent this data in the form of a frequency distribution table. Which is the most common, and which is the rarest, blood group among these students?
2. The relative humidity (in %) of a certain city for a month of 30 days was as follows:

98.1	98.6	99.2	90.3	86.5	95.3	92.9	96.3	94.2	95.1
89.2	92.3	97.1	93.5	92.7	95.1	97.2	93.3	95.2	97.3
96.2	92.1	84.9	90.2	95.7	98.3	97.3	96.1	92.1	89

(i) Construct a grouped frequency distribution table with classes 84 – 86, 86 – 88, etc.

(ii) Which month or season do you think this data is about?

(iii) What is the range of this data?

3. Three coins were tossed 30 times simultaneously. Each time the number of heads occurring was noted down as follows:

   0 1 2 2 1 2 3 1 3 0

   1 3 1 1 2 2 0 1 2 1

   3 0 0 1 1 2 3 2 2 0

   Prepare a frequency distribution table for the data given above.

4. The value of π up to 50 decimal places is given below:

   3.14159265358979323846264338327950288419716939937510

   (i) Make a frequency distribution of the digits from 0 to 9 after the decimal point.

   (ii) What are the most and the least frequently occurring digits?

5. A company manufactures car batteries of a particular type. The lives (in years) of 40 such batteries were recorded as follows:

2.6	3.0	3.7	3.2	2.2	4.1	3.5	4.5
3.5	2.3	3.2	3.4	3.8	3.2	4.6	3.7
2.5	4.4	3.4	3.3	2.9	3.0	4.3	2.8
3.5	3.2	3.9	3.2	3.2	3.1	3.7	3.4

4.6 3.8 3.2 2.6 3.5 4.2 2.9 3.6

Construct a grouped frequency distribution table for this data, using class intervals of size 0.5 starting from the interval 2 – 2.5.

6. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political party	A	B	C	D	E	F
Seats won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.

(ii) Which political party won the maximum number of seats?

7. The length of 40 leaves of a plant are measured correct to one millimeter, and the obtained data is represented in the following table:

S.No.	Length (in mm)	Number of leaves
1.	118 – 126	3
2.	127 – 135	5
3.	136 – 144	9
4.	145 – 153	12
5.	154 – 162	5
6.	163 – 171	4
7.	172 – 180	2

(i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]

(ii) Is there any other suitable graphical representation for the same data?

(ii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

8. Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table

Marks	Number of students
0 - 10	5
10 - 20	10
20 - 30	4
30 - 40	6
40 - 50	7
50 - 60	3
60 - 70	2
70 - 80	2
80 - 90	3
90 - 100	9
Total	51

Draw a frequency polygon corresponding to this frequency distribution table.

9. 100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:

Number of letters	Number of surnames
1 - 4	6
4 - 6	30
6 - 8	44
8 - 12	16
12 - 20	4

(i) Draw a histogram to depict the given information.

(ii) Write the class interval in which the maximum number of surnames lie.

10. The following number of goals were scored by a team in a series of 10 matches:

2, 3, 4, 5, 0, 1, 3, 3, 4, 3

Find the mean, median and mode of these scores.

11. Find the mode of 14, 25, 14, 28, 18, 17, 18, 14, 23, 22, 14, 18.

12. 5 people were asked about the time in a week they spend in doing social work in their community. They said 10, 7, 13, 20 and 15 hours, respectively.Find the mean (or average) time in a week devoted by them for social work.

Chapter- Probability

INTRODUCTION

Probability

- Probability is the measure of the likelihood of an event to occur. Events can't be predicted with certainty but can be expressed as to how likely it can occur using the idea of probability.
- Probability can range between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event.

Experiment

An experiment:

- is any procedure that can be infinitely repeated or any series of actions that have a well-defined set of possible outcomes.
- can either have only one or more than one possible outcomes.
- is also called the sample space.

Trail

- A single event that is performed to determine the outcome is called a trial.
- All possible trials that constitute a well-defined set of possible outcomes are collectively called an experiment/sample space.

Experimental Probability

Experimental/Empirical Probability

The empirical probability of an event that may happen is given by:

Probability of event to happen
P(E)=Number of favourable outcomes/Total number of outcomes

Coin Tossing Experiment

Consider a fair coin. There are only two possible outcomes that are either getting heads or tails.

Number of possible outcomes = 2
Number of outcomes to get head = 1
The probability of getting head
=Number of outcomes to get head/Number of possible outcomes=1/2

Rolling of Dice Experiment

When a fair dice is rolled, the number that comes up top is a number between one to six. Assuming we roll the dice once, to check the possibility of three coming up.

Number of possible outcomes = 6
Number of outcomes to get three = 1

The probability of getting three =
Number of outcomes to get three/Number of possible outcomes=1/6

Sum of Probabilities of Favorable and Unfavourable events

- When a trial is done for an expected outcome, there are chances when the expected outcome is achieved. Such a trial/event is called a favourable event.
- When a trial is done for an expected outcome, there are chances when the expected outcome is not achieved. Such a trial/event is called an unfavourable event.
- All favourable and unfavourable event outcomes come from the well-defined set of outcomes.
- Suppose an event of sample space S has n favourable outcomes. Then, there are S-n, unfavourable outcomes.
- The probability of favourable and unfavourable events happening depends upon the number of trials performed. However, the sum of both these probabilities is always equal to one.

Assignment:

1. A coin is tossed 1000 times with the following frequencies:
   Head : 455, Tail : 545
   Compute the probability for each event.
2. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.
3. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

   If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up
4. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in ₹)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000-10000	0	305	27	2
10000-13000	1	535	29	1
13000-16000	2	469	59	25
16000 or more	1	579	82	88

   Suppose a family is chosen. Find the probability that the family chosen is

   (i) earning ₹10000 – 13000 per month and owning exactly 2 vehicles.

   (ii) earning ₹16000 or more per month and owning exactly 1 vehicle.

   (iii) earning less than ₹7000 per month and does not own any vehicle.

(iv) earning ₹13000 – 16000 per month and owning more than 2 vehicles.

(v) owning not more than 1 vehicle.

5. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random
(i) likes statistics, (ii) does not like it.

6. The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.
(i) What is the probability that on a given day it was correct?
(ii) What is the probability that it was not correct on a given day?

7. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Distance (in km)	less than 4000	4000 to 9000	9001 to 14000	more than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :
(i) it will need to be replaced before it has covered 4000 km?
(ii) it will last more than 9000 km?
(iii)it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

8. The percentage of marks obtained by a student in the monthly unit tests are given below:

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

9. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00

Find the probability that any of these bags chosen at random contains more than 5kg of flour.

10. Two coins are tossed simultaneously 500 times, and we get
Two heads : 105 times
One head : 275 times
No head : 120 times
Find the probability of occurrence of each of these events.